Numberphile Podcast Transcript
Episode: Fame and Admiration - with Timothy Gowers
Episode Released October 22 2019

Fame and Admiration - with Timothy Gowers

Fields Medallist Sir Timothy Gowers discusses his career - and the role of ‘begrudging admiration’ in mathematics.

Sir Timothy Gowers webpage at Cambridge

Timothy Gowers blog

And his Twitter

The Fields Medal

Terminator 2

The unconditional basic sequence problem

Szemerédi's theorem

Tim Gowers International Math Olympiad results

The Fields Medal with Cédric Villani - Numberphile

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Brady Haran [BH]: Ready to open your heart to me?

[gentle music fades in]

Timothy Gower [TG]: Well, we’ll see. [laughs]

BH: [laughs]

[music fades up]

BH: Today's guest is the English mathematician Sir Timothy Gowers. [music continues] Among his numerous accolades is the Fields Medal he won in 1998. Well, it's a medal he pretty much hasn't seen since by the sounds of it, but we'll come to that later. [music continues] I visited Professor Gowers at his office in Trinity College at the University of Cambridge.

[music fades down]

BH: Where are you from actually? Where were you born? [music continues] You sound very English.

[music fades out]

TG: I am… pretty English.

BH: Yeah?

TG: I was born in a town called Marlborough in Wiltshire and grew up in London.

BH: Did you grow up with mathy people like were your parents mathematical?

TG: They were musicians but my father was always interested in maths and did maths up to… A level.

BH: And were you like one of these prodigy boy… were you like… was it

obvious you were gonna be a mathematician?

TG: Um, I was… quite… uhh… I think it was sort of… when I was quite young, and I sort of picked up music quickly, so… [groans] that was quite a big thing. And at school I was one of the good ones, but not sort of far and away the best person at school. Particularly in maths…

BH: Right?

TG: …but in general I was sort of reasonably good all around there and… maths was always gonna be in the mix somewhere.

BH: Did you want it to be in the mix, like if I'd spoken to you as a boy and said what do you want to grow up? What would the answer've been?

TG: When I was really young I think I wanted to be a doctor because there were doctors in my family, but I think I wanted to be a doctor with a beard, and I don't know exactly why that…

BH: [laughs]

TG: …detail was… so I'm actually neither.

BH: Professor Gowers is clean shaven for those who don't know what he looks like.

TG: [laughs]

BH: Very well clean shaven.

TG: I suppose I did have a PhD so I was a sort of doctor for a while.

BH: A doctor with a beard? Did you doctor had a beard? Like where did that get into your head?

TG: I don't know. I think maybe some… there may have been an ancestor. My mother's father was part of a long line of doctors in Marlborough actually. So that may have had something to do with it. I'm not quite sure. Anyway, my father was a composer and I think I had some aspirations in that direction at a certain age, but [sighs] maths was not by… not as by a very long way but certainly I think I would say usually my favorite subject at school and I think that was partly because of less learning to do, you just… once you understood something… course I realized later that that was only… up to a certain point…

BH: Yeah. [laughs]

TG: …then after that point there's a lot of learning to do in maths.

BH: Yeah.

TG: …but… [pauses]

BH: But it was like a path of least resistance at that point?

TG: There's a bit of that, yeah, so I feel that there's… there's a slight sort of element of laziness to my… liking of mathematics.

BH: Well did you like music, or was that because your parents were musical?

TG: I very much liked music.

BH: Alright.

TG: And… music was a big part of my childhood and I sang for example as a

chorister in King's College Choir, here in Cambridge, which was a… left its mark somehow. And I learnt the violin and the piano and…

BH: You're still musical now?

TG: Yes I don't not very actively but… not completely inactively.

BH: Do you still sing?

TG: No that stopped… when I got to about the age of eighteen my broken voice wasn't something that really should be inflicted on the world.

BH: Okay. [laughs] At what point then in your school years did the switch happen where it was like, okay, I'm gonna be a mathematician, I'm gonna take this further?

TG: Well quite late, because I certainly… basically at each stage of my… childhood and early adulthood there would be… I'd be doing something. I'd be doing A levels or a degree or a PhD and there would be something just over the horizon which would be the next stage and at each stage I wanted to get to the next stage so by… when I was doing O levels as we did then I wanted to do A levels in mathematics, and when I did A levels in mathematics I wanted to do a degree and when I was doing to degree I wanted to do a PhD, and when I was doing a PhD I wanted to get a research fellowship afterwards. And each stage…

BH: But…

TG: …I cleared the hurdle, so to speak.

BH: But at O levels you presumably you did English at O levels as well?

TG: Oh yes, so it was…

BH: So there was a…

TG: …the full range.

BH: So there was a next stage for English as well? You could've done A levels English, you could've a degree in English.

TG: Right.

BH: Why was math the one you wanted to jump the next hurdle of?

TG: Well at that point it was because what I was [hesitates] said I mean… actually I… think I could have enjoyed English but I think it maybe it wasn't just a sort… the fact that in maths you didn't have to do much learning… it was also that and maybe this is a… yeah… the tasks were sort of more [sighs] well-defined. So at school… nowadays I really enjoy writing, effectively essay writing in the form of blog posts and things like that, but at school I really didn't like it because… the task was somehow… if we take something like a history essay that we were told to, you know, we're given a title and had to write it, but writing a history essay meant looking at one source, which would be a textbook and… sort of rewriting a small chunk of it in our own words and not trying not to stick too closely to the textbook but not deviating too ridiculously from it and it was a silly exercise.

BH: Hmm.

TG: Nobody had really stopped to say, what a good essay was and one didn't have something, I mean I find writing… an essay is a whole lot easier if you have something you want to say in the first place, whereas if your churning out an essay as part of your school work you've been told what to say and so it's not something that's coming within that you're keen to explain to people. So I didn't

enjoy the humanities in the way that if I went back now as an adult, perhaps I would, so that was another reason for… the tasks that one had to do in maths were much more clearly defined and… but also I think another aspect of it was the… [pauses] genuine intellectual excitement of… things like… learning calculus, I remember when I first did that and a whole lot of problems that would've seemed completely impossible… suddenly became possible and when I saw a car accelerate somehow my whole perception of that would be completely different once I knew about derivatives and integrals and I remember, I really remember feeling that the world had become a clearer place [laugh] after I learnt calculus and that was very exciting.

BH: A lot of people I've spoken to who are mathematical have spoken about how when they first got to university that's when they got excited about mathematics because they started learning about the creative side of it and proofs and all these new things they could do. It sounds like that came a bit earlier for you though? Like, just knowing the power of the tools was what excited you, not even the creative side of it yet.

TG: Yes that's sort of true but also I did have various extremely enlightened and interesting teachers. And in particular when I was a teenager I had a teacher, he was called Norman Routledge who didn't just teach us the tools and so to speak, the boring stuff, even if some of the boring stuff is very interesting, but also… gave us problems that had twists and required creativity and things. And also towards the end of my school days, the Maths Olympiad became quite an important thing and those were definitely problems where you needed a bit of creativity.

BH: How did you do in the Maths Olympiad? I forgot that you'd done that.

TG: I made the British team in... uh… 1981.

BH: Right.

TG: We went to Washington DC. Uhh…

BH: How'd you do?

TG: I got a gold medal.

BH: Gold medal!

TG: It was an interesting one actually, I actually got… full marks but that sounds better than it was because… they were introducing a whole lot of new countries and I think they made the questions a bit easier.

BH: Right.

TG: And so rather a lot of people, I think about twenty-five people got full marks so it wasn't…

BH: Right.

TG: …it's not what it would've been in another year.

BH: There's also this… I don't know much about the Maths Olympiad, but the little bit I do know is they then have these extra medals they give for students that come up with like unexpected proofs and solution.

TG: Right. Yeah I didn't get one of those.

BH: [tsks] Ahh.

TG: [laughs]

BH: Nevermind. You've made up for it since.

TG: [laughs]

BH: [laughs] Have you still got the gold medal? Do they give you an actual medal… like?

TG: I… actually can't remember. I remember they gave us a… programmable calculator. Which was quite a nice thing to get.

BH: Yeah?

TG: Which I had for a while until it… eventually died.

BH: Right. [laughs] Did you keep it anyway, like as a memento on a shelf? Or did you toss it in the bin?

TG: It's not something I would've thrown away… I have a sorting that's intermediate between keeping and not keeping which is putting in a place, a storage place, in the house that's full of [laughs] a lot.

BH: Right. [chuckles]

TG: So I think it must be there somewhere but I have no idea where exactly.

BH: That's my one day, I'm gonna go with you and find that.

TG: [laughs]

BH: [laughs] That's on my list now. [laughs] So Math Olympiad, so you were clearly very good at mathematics.

TG: At some point my father said to me, that… one thing that you can do… he'd been interested because he'd saw that I was interested in maths, he talked to one or two people he had met who… and said what kind of careers are there in maths and…

BH: He wasn't shepherding you towards music? 'Cause I mean you say…

TG: No, no, no.

BH: …your father was like not just a composer was he? He was like quite a successful one?

TG: Yes he was quite… well known in certain circles so to speak.

BH: Yeah. But he wasn't pushing you towards music? He was fine with…

TG: Not at all, no, I think he took the attitude that many musicians would take which was that if someone has a… a real calling that they can't not be a musician then but they better be a musician but… [laughs]

BH: Yeah… [laughs]

TG: In a way I was… by doing maths I was following a path that he would in another life perhaps have liked to follow himself so I think I was in a way doing something that he was… I'm not saying I felt pressure in that direction either but…

BH: Yeah.

TG: You know, I was… it'd please him that I went in that direction.

BH: As in mathematics or as in just like academia and doctorate…

TG: No not so much academia, in fact he… himself could've been an academic musician, 'cause he did very well at academically when he was at university and it was fairly clear that he could've taken that path but he decided to take the riskier path of leaving and trying to make it as a composer… which…

BH: So anyway, he planted the seed, you know, you could go into academia, you could be a…

TG: Actually it was almost the opposite so he was saying that from the people he'd talked to… [pause] he got the impression that it was very good to do mathematics because it's always much easier to turn away from mathematics at some point to go back into mathematics and so however much of an applied type thing you want to do. Sort of… if you stay pure for a long time then at some point you can move out from that so it was a good… it was keeping one's options open.

BH: Right.

TG: But he just mentioned at that point that one option if you carry on with maths, so to speak forever, is being… going into academia, but that's an extremely hard thing and you have to be so to speak right at the top and so… I assumed at that stage when I was at school that I had no chance of that or that was very unlikely but when the Olympiad thing happened that was a sort of semi-objective measure and I sort of realized that at least it was something that might be a possibility. But I would say even at that point… I didn't really have much conception of what it was to be a mathematician as a job.

BH: Right.

TG: What was research was like. If somebody had said to me at that stage, what is research in maths? I couldn't… there's a question that people often ask,

how can you do research in maths, surely it's all been worked out a hundred years ago and… once one is a mathematician one realizes that… that's just so far from being the right way of looking at things and that every time you solve a problem you accidentally create five more problems and… the subject is just completely inexhaustible.

[gentle violin music]

BH: Where did you go to university then? Where did you…

TG: At Cambridge.

BH: Right?

TG: So… and… uh… but even as a beginning undergraduate I didn't really have much of an idea of what… mathematical research could be like, in fact possibly not even starting a PhD, 'cause a PhD one has this… slightly… one feels a bit daunted that… how could… I, who've only just graduated, possibly solve a problem that these brilliant mathematicians over the last few decades have not manage to solve?

BH: Why did you do it then? Why did you start a PhD? If you…

TG: Because… I could sort of see empirically that people did go into PhDs and did solve problems. [chuckles]

BH: Okay, right.

TG: Get their PhDs.

BH: You think if they can do it, I can do it?

TG: Well, that I have a chance of being able to do it.

BH: Yeah.

TG: It's just that I couldn't really see how it might be possible and I suppose the things… the two things that I didn't appreciate as much as I, you know… at the time where one, just how much the subject expands and so there are… all the time it expands, and so there're always problems that… they may be quite hard but just because they're open problems it doesn't mean that all sorts of top mathematicians have worked on them, sometimes they're just haven't… people haven't got 'round to those problems or they're questions that arise naturally out of a paper but… it doesn't, you know, maybe one or two people've thought about it but… and then the second thing I suppose I appreciate now particularly when I get to my stage is… my career stage, is just how much… the experienced mathematicians, how much… how many demands there are on their time which maybe stop them doing a certain amount… a certain kind of thought. There's a certain sort of thought that you can do as a young mathematician with lots of time which is just really worrying away at something for ages and ages and being completely focused on that thing and sometimes that's what's needed to solve a problem you just need somebody who will worry away at it until eventually the difficulties… get chipped away and they finally end up with a solution and sometimes the time investment that's need… and the single mindedness that's needed for that is something that you just don't really have the opportunity for later in the career so there was also sort of a young persons problem, and I think older people are more likely, it's not… obviously this is over simplifying a lot and there are exceptions to everything but older people will tend to look a bit more for something that's a bit of a quicker gain given the experience that they've built up over their career.

BH: Right.

TG: So that ends up… often solving things that younger people might've

found difficult but it's a different style of question which leaves open the other style of question [laughs] that the younger person can…

BH: But I mean professor, you're clearly quite a humble person but it would be pretty hard to deny you've become quite accomplished as a mathematician and won all these awards and have all these honors so obviously you're like a top top mathematician. At what point in your career as a younger mathematician like at university did people start saying to you, Tim I think you could be really good at this, I think you could, you know, I think you could be one of the really good ones. What point were you getting that some kind of stuff said to you? As an undergraduate or does it happen when you become a PhD?

TG: Definite… no not as an undergraduate and I think I didn't… as an undergraduate… stand out as somebody who was obviously destined for… an academic career. When I finished my PhD I got a research fellowship at Trinity College Cambridge. That is… probably… as for most for people that will be…

BH: A pretty big deal?

TG: More or less a guarantee of a career in academia unless you just don't do anything during your fellowship.

BH: Yeah [laughs]

TG: So at that point I had sort of got it made in a certain way. In the sense that my career prospects were looking good.

BH: Can you give us some idea about your area of research that you were doing at this point?

TG: Well I started out in an area known as the geometry of Banach spaces. So a Banach space is… it's a bit like a generalization of three dimensional space to

infinite dimensions. It doesn't… it actually they don't have to be infinite dimensional but large finite or infinite dimensional. But there's also a twist which is that in three dimensions if you want to work out the distance between two points the most common way of doing it is to look at the… just add up the squares of the differences of their coordinates and take the square root, so that's using Pythagoras' Theorem. But that's only one form of distance, maybe there's something… a distance that people often like to talk about in two dimensions actually called a Manhattan metric, so if you're going from one part of Manhattan to another, the distance as the crow flies would be… the one you calculate using Pythagoras but the distance that actually matters is the one where you have to always go horizontally or vertically because that's just 'cause of the grid plan of Manhattan.

BH: Hmm.

TG: And in general… there turn out to be many different sensible ways of putting a distance onto a high dimensional space. If you then look at the set of points that are of distance one from the origin, it turns out that you get a convex body, a shape that is convex, so that means any two points if you look at the line segment… any two points in the body, the line segment must lie entirely within the bodies, there are no sort of dimples on the side.

BH: Right.

TG: That would allow a segment to pass out and then back in again.

BH: Like a sphere in three dimensional space?

TG: So a sphere is a good example of a convex body and so is a cube… but… something like a torus would not be convex or a sort of dart shape in two dimensions… anyway is not convex. So because of this correspondence between notions of distance on space and convex bodies you get… that's why the word

geometry comes in, so the geometry… there's a study of these Banach spaces, these notions of distance on high dimension space, has a geometrical aspect. It's called the geometry of Banach spaces. So I worked in that. It's also a branch of a… wider part of mathematics called functional analysis. So I was an analyst, but of a somewhat combinatorial kind and then over the years I moved more into that combinatorial side became accentuated and then… the analysis became lessened. I've sort of gone from a combinatorial flavored analyst to an analysis flavored combinatorialist. [laugh] I think that would be the way I would… describe my research interests. So combinatorics I've always liked because maybe you'd say it's the branch of maths that's closest to Maths Olympiads you have [laughs] sort of… problems that can be… where the question can be understood very quickly and easily but finding the solution is hard and requires a lot of ingenuity.

BH: When you started your degree, obviously you had the whole buffet of mathematics before you, what pushed you in that direction? Was it a person? Was it just an enjoyment… a natural affinity for the subject? Why did you end up these Banach spaces?

TG: Well it's… funnily enough it's a continuation of the instinct that drove me into maths in the first place. Well, it's two things. So you asked briefly whether it was a inspiring person and the answer was yes, there was a person called Béla Bollobás who became my research supervisor and had been my director of studies as an undergraduate who's a very charismatic and brilliant mathematician and… I had a friend, a good friend, Imre Leader who was his student and really clearly enjoyed it. And that was what… what Bollobás was doing at that point. He's a combinatorialist but he was also working in Banach spaces at that time. So that's one reason, a particularly influential person. But another reason was that there are some branches of mathematics where you have to learn a whole lot of machinery and really get very up on the literature before you can realistically do serious research, a good example of that would be algebraic number theory, the branch of maths that goes into the proof of Fermat's

Last Theorem for example. Combinatorics is slightly the other extreme, you can just dive in and start trying to solve the problem straight away.

BH: Right.

TG: There are bits of machinery and useful tools, definitely, like in any branch of maths but you don't have to master them all in advance you can just pick them up as you need them. So I found that quite appealing. The same sort of laziness thing [laughs] I just wanted to get on and…

BH: Yeah.

TG: I'd had enough after what we call part three mathematics where that was the amount of learning I had to do there was just huge and I just wanted to… stop all that and just think about problems and try and solve problems and just at that point I really wanted to solve just any unsolved problem. The idea of being the first person to solve a problem was just so thrilling.

BH: That was… you had a hunger for that, like a hunger for that kind of side of…?

TG: Yeah, the sort of… yes… I suppose if you ask my why, that's an interesting one. Um… part of it was a genuine interest in the subject. But there was also this idea of sort of wanting to be the person who solved such and such a problem.

BH: So is that almost like… like a fame or an ego like a...

TG: No, I think… I have to admit I think there is that side to it.

BH: There is a degree of wanting to be… the person who did it, the person who found it? Like the Neil Armstrong or the…

TG: Yes I'd… I think that… wanting to get a certain amount of… fame and admiration… well as somebody once put it, begrudging admiration of a handful peers or something [laughs]…

BH: Right. [laughs]

TG: Anyway… is a very important motivation for many mathematicians.

BH: Yeah.

TG: Whether they admit it or not, I don't… [laughs]

BH: Yeah.

TG: And it's an interesting aspect of the subject that has its problematic side. So I've often felt that the most efficient way of doing mathematics might well to be… be to be far more cooperative and to… share one's ideas as soon as one has them and so on but then you have that balance that against taking away one of the power drivers, I mean… you've gotta have something that's gonna make people prepared to put in the effort…

BH: Yeah.

TG: To solve a hard maths problem, which will then be understood by virtually nobody in the world. [laughs] So…

BH: [laughs] Yeah.

TG: I think unfortunately this sort of ego boosting side of things is an essential part of the discipline.

BH: It's part of the compensation package.

TG: [laughs] Something like that yes.

BH: Yeah. So that kind of model of the… the Andrew Wiles hiding in the attic and not telling any what he's doing for fifteen years or something has its merits because otherwise that work just wouldn't get done in the first place.

TG: Yes it does. I mean I feel that though… I feel uneasy about that story being… it is fantastic that it happened but… it's some… romantic stories in mathematics is a danger that they can be a little bit harmful because they're really very very untypical and I would never… any research student of mine, I would say don't… what ever you do [laughs]…

BH: [laughs]

TG: …go into an attic for seven years without telling anyone what you're working. It's just… it was a massive risk that paid off for Andrew Wiles. He must've I suppose… sensed that… he had a reasonable chance of success but… I think for everyone of those you can find…

BH: A wasted life. [laughs]

TG: There are examples, I won't mention any names but there are examples of people who just decided they wanted to do something a bit like that and it didn't end well.

BH: Do you remember your first flag planting Neil Armstrong moment? The first time you felt like you'd found something new and significant? Your first big moment like that?

TG: Um… it came gradually, so the first time I did anything new… I don't

remember the precise moment but the thing that I did was just a little tweak to an existing argument that… improved the… answer at the end. Improved the bound, but not to the best bound you could possibly get and… I remember presenting it out at a seminar where one of the experts in the area was present who didn't seem, you know, it's sort of, okay. [laughs]

BH: [laughs] You didn't get the response you were hoping for?

TG: Yeah but in a way that spurred me on a little bit and so I worried away. I mean I did what I was talking about, worried away at that particular problem and eventually did get, by a much more complicated argument, an almost best possible answer for that particular problem.

BH: Did that person then get impressed?

TG: Uhh… I don't know. I think he… well he certainly would've been likely to have been more… more interested but… I don't… but somebody, there was another mathematician who was very supportive to me at that stage of my career, who definitely liked this result. So that was helpful. But there's… another thing that I recall… I mean, it was the first… the thing that I really think of as important first was not so much the first… serious piece of work that I did. It was the first time I solved a problem that was something that I'd kind of seen… [pauses] at some point and dreamt about so to speak, I mean I've… as it were… [sighs] as a young… [pause] a young child seeing Fermat's Last Theorem and thinking it would be amazing to solve that, it was a bit like that except it was local to this area. There were a couple of problems…

BH: Right.

TG: …that were… big problems in the area that were mentioned as open problems in a… one of the basic textbooks in the area.

BH: Right.

TG: Lovely problems. And… I suddenly at some point realized that I had an idea that gave me a serious chance of solving it as I worked on it absolutely flat out [clears throat] over one summer.

BH: What was it called?

TG: It was called the Unconditional Basic Sequence Problem. So I solved that with a… complicated counter example.

BH: So this wasn't a proof, this was almost just like a… this is different to a proof?

TG: It was a solution to a problem, and the problem said, is such and such always the case… does every Banach space have such and such a property.

BH: Yeah?

TG: And the answer was no, here's a horrible Banach space that does not have that property.

BH: So you have kind of proven something, by counter example?

TG: Yes you have, yes. No, no, that's…

BH: Yeah, yeah, right.

TG: …it's just… not… maybe sometimes counter examples slightly sort of close off an area and sort of had a little bit of that, although there were some positive outcomes of it as well. But anyway, the story was complicated because not long after I did it… I heard… that another mathematician had also done it.

Actually very slightly after me but definitely independently of me, and I was so shocked when I first heard that that I realized I just couldn't do anything… so I went to the cinema, it's the middle of afternoon [laughs] I went to the cinema instead and watched Terminator 2.

BH: [laughs]

TG: [laughs]

BH: It's a good movie.

TG: Yeah. It sort of…

BH: [laughs]

TG: …took my mind off it for a… couple of hours and then I came out again, boom, it was back in my mind again.

BH: Why did that worry you though? You knew you'd done it, and you'd done it marginally first, so why was this such a blow to you?

TG: Well in retrospective it shouldn't've been but just at… the time… I sort… it felt like losing half the…

BH: Oh right.

TG: …half the credit for this thing and…

BH: Okay.

TG: …not being the person but it was all sort of… actually what happened was we then wrote up a paper together and then there was a follow up paper

that was… had some other results and I now realize that when this happens, which happens quite often, when someone solves a problem and somebody else solves it independently, you don't actually… people sort of say, wow, you've solved the problem. Even if it's not independent, even if two people solve it jointly, I feel that they get individually about as much credit as if they [laughs] solved it…

BH: Right.

TG: …individually but at the time I was just sort too inexperienced… but I was… the shock and disappointment passed reasonably quickly.

BH: After Terminator 2 basically. Put's everything in perspective when you see the world being annihilated. [laughs]

TG: That definitely helped but it was immediately afterward. [laughs]

BH: [laughs]

TG: But then I sort of went on and did other things as well and… so that is also a bit like if you… a relationship breakup, there's nothing like another relationship [laughs] to help.

BH: Yeah. Yeah.

TG: So it was a bit like that with research as well. So I've subsequently, I mean everybody, I mean every mathematical career has disappointments in it of this kind, not necessarily precisely that kind but maybe a problem that you've been working on for a long time gets solved by someone else and… or… yeah well there are the various different ways but… [swallows] you sort of get used to that as being… part of… mathematical life… and just… the solution to it is just to pick up… pick yourself up and… keep going and do something else instead

and…

BH: Yeah.

[gentle piano music]

BH: So you won the Fields Medal and I know the Fields Medal is awarded for like a body of work, almost, like a consistent. Is there kind of one thing, was there a pinup thing? Like what would the thing that they chip into your tombstone or something? What was your big one?

TG: I think in my case it was not just one big one, it is for some people, for me it's more like a big three or four things. So there was the counter example I've just talked about in Banach spaces and then there was also a theorem in Banach spaces that proved… solved positively a question that Banach had himself asked, which was I suppose one of my headline results. And then there was also…

BH: Did you get your name on it? Is it called the Gowers Theorem or anything or…?

TG: Not really, that's not referred to in that way but there is something related to it called the Gowers Dichotomy which was part of the proof.

BH: Okay.

TG: [laughs]

BH: That'll do.

TG: Sometimes no, though anyway.

BH: Okay.

TG: And that's quite in a way that's quite a nice one.

BH: You've got your own dichotomy, not many…

TG: Exactly, yeah.

BH: …people've got that. Everyone's got a theorem. [chuckles] You've got a dichotomy.

TG: Yeah, if someone gets a trichotomy then I'll be envious but…

BH: [laughs]

TG: [laughs] And then… but around about the time that I got it, the Fields Medal, I was… had started my move towards more combinatorial things and in particular a branch that's now known as additive combinatorics. And so perhaps a bit oversold the results, if looking back now the thing which I'd be remembered for most out of those… results would be for a new proof of a result called Szemerédi's Theorem. It may sound a bit strange that finding a new proof of an existing theorem should be a thing that stands up but it did do something that hadn't been done before, it gave, a sensible quantitive bound for this particular theorem, which hadn't been done before, but also that the methods that went into it had been quite influential and lead to other developments. So and then that sometimes happens that a… that… a proof can be more important than the result that it proves and I think that's a little bit the case here, that… the ideas that went into the proof are more… the theorem is not something that you'd necessarily… occasionally it can be directly applied to another problem but that's less usual than the ingredients that went into the proof being applied to another problem.

BH: Do you remember how you found out that you'd won the Fields Medal?

TG: Uh… yes I do. I was summoned by my then head of department, John Coates, and so was someone else in my department. So he said can you come and see me and so we both went it.

BH: What did you think it was? Did you think you were in trouble? You'd been raiding the stationary cupboard? [chuckles[

TG: I'm wasn't quite sure although… [pause] John Coates had encouraged me to apply for a chair in the department way before I would've thought it was reasonable to apply for a chair. So it was already an exciting time, and I got it, I knew by that stage that I'd got the chair which came as a big surprise. A named chair it was… and I thought I was much too… young in my career for that even to sort of… [pauses]

BH: Yeah?

TG: …not to sort of insult the department [laughs] to imply…

BH: So you knew you weren't in trouble 'cause you were in the good books at the moment.

TG: Yes.

BH: Yeah? [laughs]

TG: And then he said… well I've got some good news that you've both… got… the Fields Medal. So this was completely out of… it was very unusual for… [pause] I'm not sure whether it's happened before that two people from the same department have got them at the same time. So because there were two of us it sort of didn't feel… I didn't think… I'm trying to remember I don't think I suspected that he was about to say.

BH: Oh okay.

TG: And I don't think I knew that he would have any idea about it anyway.

BH: Yeah, like the only people I've spoken to about it, found out directly from the… that they'd won the Fields Medal, but you found out kind of like… second hand?

TG: Well… it wasn't totally second hand. I mean he was on the committee.

BH: Oh, okay. [chuckles] Right.

TG: As I subsequently learned.

BH: So it was sort of part of his job to tell you, as well?

TG: Yeah.

BH: How did you feel? You're sharing credit again. You're having to share it with someone else.

TG: No, no, that was okay.

BH: That was okay. [laughs]

TG: That was… actually in some ways I think that was quite good because then, one thing we had to keep a lid on it until the International Congress which was several months later.

BH: Hmm.

TG: Yeah, I can't remember it was probably January or February that I was told and then August when finally it became public.

BH: Right.

TG: But I think partly because it was two of us and the sort of level of… celebration and fuss that was made, once it did become public within Cambridge and within Trinity, he was also at Trinity, was perhaps greater than it would've been if it had just been one person.

BH: Yeah. Yeah. Who was that by the way? Who…

TG: Richard Borchards.

BH: Right?

TG: Who's at… at uh… Berkeley.

BH: Where is your Fields Medal? You don't know do you? For sure? [laughs]

TG: Well, I kind of… I'm know to within of ninety percent… certainty.

BH: [laughs]

TG: So I… at the time I was a bit worried about keeping something that's quite chunky and made of solid gold to sort of keeping it in a drawer somewhere in case the house got burgled or whatever.

BH: Hmm.

TG: So I decided to get a safe deposit box at my… local bank. And… uh… I've put it in there and I haven't seen it since. I haven't sort of checked and the bank

has moved branch.

BH: Right? [laughs]

TG: So I did get a bit worried about it a few years ago, but I wrote to them and asked, you know, I haven't really heard anything about this safe deposit box so I thought that I… got charged a certain amount per year, I've not really seen anything connected with it on my bank statement…

BH: Right.

TG: …for a long time.

BH: [laughs]

TG: I'm a bit anxious. And they said, oh well no if it was in the safe deposit box there it would've been moved to this branch so really there's nothing to worry about.

BH: But you haven't gone and had a look to see?

TG: But I haven't actually got 'round to… yeah.

BH: When did you last see it with your eyes?

TG: I suppose it must be about… uh… getting on for twenty years ago. [laughs]

BH: Oh! For goodness sake. That's the other… when we go and find your calculator, we're also gonna go and get the medal. [laughs]

TG: Yeah, well, maybe we should.

BH: [laughs]

TG: [laughs]

BH: That's your next Numberphile video. [laughs] To dig up the relics. We're go on an Indiana Jones adventure. [laughs]

TG: That would be exciting, yes.

BH: Yeah?

TG: [laughs]

BH: [laughs] So among, well, you know, you've obviously won all sorts of things and but another honor you've got that I'm a little bit fascinated with is a Knighthood. Because you are… Sir… are you Sir Timothy? Or is it Tim?

TG: Sir Timothy I suppose.

BH: Sir Timothy. So… who did the sword for you? Who knighted you?

TG: Oh I had… I got the Queen.

BH: Oh! Yes!

TG: [laughs]

BH: That's like the… well done. [laughs]

TG: Hit the jackpot there.

BH: Yeah amazing. What was that like? What did she say to you? Or are you allowed to say what the Queen says to you?

TG: I'm allowed to, whether I can remember it's…

BH: [laughs]

TG: …only it's that… you know if you've got your gong for… training horses or something then the Queen's sort of very happy but…

BH: [laughs] Twenty minute conversation then.

TG: If you turn up as a mathematician…

BH: [laughs]

TG: You can see the look of sort of terror on her face, what on earth am I going to say?

BH: [laughs]

TG: And… so, well I don't know whether I broke protocol but I thought… she did look as though she was worrying about what she could possibly say so I thought I might… help her out a little bit so I said… I don't know exactly how I put it but… roughly it boiled down to, we last met when you came to Cambridge to open… um… the new maths faculty there. And so she looked sort of [laughs] relieved and said yes and something like… is it going well or something? [laughs]

BH: So you broke the ice for her?

TG: Yeah. Before… once it's happened protocol then dictates that you aren't

allowed to turn your back on the Queen, so you have to walk backwards away from her before turning and going… returning to your seat so I sort of headed backwards and [chuckles]

BH: For people who don't understand my excitement by the way, when you get a Knighthood you can… it could be conferred upon you by the Monarch herself or it could be like you know Prince Charles or someone else, the Duke of Cambridge and… so… I always think getting the Monarch themself is like… top top level…

TG: Yeah so…

BH: So you were lucky. You're the first person I've met with a Knighthood who also got the Monarch. Everyone else I've met got Prince Charles, so, well done. [laughs] Luck of the draw, or were you special?

TG: No, no, I think I was just… I think Prince Charles was doing some of them around that time and I just…

BH: Yeah?

TG: I think well, I'll be even luckier now, I think she does, the Queen does fewer and fewer of them as she gets older and older 'cause she has to stand for a long time to do it and…

BH: Yeah, yeah. Do these mean much to you? Do Fields Medals and Knighthoods and things like, you know, what do they… how do you feel about them?

TG: Uh… the Fields Medal is a complicated one. I mean I think you can't… unless you're… um… a really… one hundred percent shoe-in for the Fields Medal which I wasn't, but, if you're somebody like… Peter Schulze recently,

where everybody sort of knew for years that he was gonna get a Fields Medal. I think you feel a little bit of sort of imposter syndrome because… especially as one progresses you just see more and more younger mathematicians doing completely incredible things that you couldn't've do yourself and you sort of… you feel more acutely that you're just one person in a large community of very good people. So… in a way you start thinking well a Fields Medal doesn't mean all that much. But… that's something you can only… it's much easier to say if you've got one. [laughs] So… [laughs]

BH: [laughs]

TG: I don't really mean that. I mean I… it has meant, in a way, a huge amount that I sort of relax about whether I'm gonna be remembered after my death for any length of time. I'll be on that list and that's been… I think that does… um… change one's…

BH: Is that motivating? Or does it make you rest on your laurels?

TG: No. Well that is a very… I mean you've may have… be aware that there was a some literature about that and somebody did a study of… the subsequent mathematical output of one, Fields Medalists and two, people who were for various reasons thought of as strong candidates for Fields Medals who didn't get them.

BH: Hmm?

TG: There's quite a strong effect that the people who didn't quite get them have produced more papers and things than the people who did.

BH: Were you part of that study? You must've been.

TG: I suppose I was, yes.

BH: Oh.

TG: Um… I've never produced a huge number of papers, I mean I've been a rather… my total number of papers is not that… high for somebody of my age, but… one of the things that was suggested was that if people who've got Fields Medals feel freer to do slightly different things, it's not that they become… stop working altogether, although maybe that does happen in some case. But that they just might have other projects in life and when they're not any longer struggling for recognition in mathematics they can afford to do that. So in my case that has been… a… the case… a little bit. So for example I took on editing a book called the Princeton Companion to Mathematics, which was an absolutely massive task, it took, I would estimate half of my working time for about five years or something like that. So that was a big chunk to take up, but it was a project I believed in and possibly wouldn't… I probably wouldn't've actually been offered to chance to do it if I hadn't… been a Fields Medalist but I also might well have thought I can't do that, because my research is too important.

BH: Yeah.

TG: And my… I mean my research definitely did suffer while I was doing that to some extent. It freed me up… I've got interested in… Artificial Intelligence, and automatic theorem proving in mathematics which… are things that I've perhaps taken a bit more seriously and devoted more time to than I would have felt I could if I'd… not… somehow become fully established as a mathematician.

[gentle piano music]

BH: Can I ask you what research looks like? I'm always fascinated by the idea of mathematicians doing research, you know, oh I've got a week, I'm just gonna do it, spend a week doing research. What does that look like? Do you sit at this table here with a piece of paper and a pencil? Do you go for a walk or ride a

bike? Or do you like… like how do you… if you've got time set aside to have an idea [chuckles] how do you have an idea?

TG: Mhm. It's changed a bit over the years, so when I was… when I started out it was just paper and pen and then I would try and have ideas and [sighs] when I did have an idea I'd write them up using… pen and paper and only when I was done would I then… type it up because typing was so much less convenient. Now I use a screen much more and I also now almost all my research these days is in collaboration. Mainly with my research students so one thing that I like to do with research students is have a private blog where we all write posts with our ideas and comments on posts.

BH: Hmm.

TG: Little just nuggets of thought. So actually that works quite well because say I've got… sometimes it's quite nice to have a little time limit, so supposing I've got twenty minutes before lunch and I think, I've just got to do… make an iota of progress between and now and lunchtime, I can sort of look at the blog, what's my thoughts just then. It doesn't have to be solving or answering a question that… we'd been struggling with for months, it can be just one idea, maybe… [pauses] I think maybe the mysterious thing is when you read a solution to a problem it's just all there as sort of one unit and everything seems to depend on everything else and so… you might ask, how can all that come into your head at once?

BH: Hmm.

TG: And the answer is it can't come into your head at once, you have to break it down. And this breaking down process is quite an interesting one but the fundamental activity is, I would say to following… you've got a question, you don't know what the answer is, you think there's pretty well no hope of just seeing the answer to that question, so you ask another question and the other

question will be something that is… something that you judge to be easier to answer and something that will shed light on the first question so once you've answered it, your task of answering the original question will have become just a little bit easier.

BH: Hmm.

TG: And if you can do that enough. And that it may… this is an iterative process so your first question may be very hard, you may reduce it to an easier question or not necessarily reduce it but you may come up with an easier question that feels useful in that way, which again you can't answer and if you can't then, you try and find an even easier one.

BH: Yeah.

TG: And there are various different ways of generating easier questions so you might for example generalize the first question and see whether you really needed all the hypothesis.

BH: Hmm.

TG: And then answer may be… actually the question becomes cleaner and easier to answer once you've generalized it. Or it might be that once you've generalized it, you find a counter example which tells you the hypothesis that you got rid of was in fact necessary and would form an integral part of any positive solution to the first question. Once you know that it's sort of… your a bit better on the scent of how to answer the question because you know it's somewhere along the line you've got to use such and such, so when you're trying to solve a problem you're sort of searching for the particular chain of reasoning that will lead to the solution and what you're trying to do when finding it is not necessarily to find it straight off but just to narrow down the search for this chain of reasoning. So you say, well I don't know how to prove it but I know that any

argument that I'm gonna come up with that works is going to have to have these properties and… the more you can… get those properties… to narrow down the chain of… what the chain of reasoning can look like, the easier it becomes to find the chain of reasoning.

BH: Does that mean, in your case at least like the best proofs and the great discoveries that you've been responsible for haven't had a moment where you suddenly every… suddenly something switched and you were like, oh my goodness! Why didn't I think of that before? Is it just like a series of… tiny versions of that and it all gets muddled or do they still sometimes have this moment where you're walking across a bridge or standing in the shower and it's like [gasps] something changes in your brain? It's like, oh… that's it, that's the thing.

TG: I have had a few moments like that, it's a sort of mixture but quite often… it's … [pauses] been a case of needing… so… there are one or two examples where I think I've… I could pinpoint the moment where I went from feeling as though I really was quite a long way from having a solution to some sort of thing that maybe it's… confident after that moment… that… once I'd dotted all the eyes and crossed all the tees I'd have a solution.

BH: Hmm.

TG: Much more common is to need five or six… moments like that but smaller moments so you've done a lot of sort of reducing to easier questions which are quite hard and then you have a little sort of mini ah ha moment and solve one of those questions and then another one…

BH: You can move back up to the higher question?

TG: Yeah something like that. One of the great best compliments I ever had was… giving a talk on the result I mentioned earlier Szemerédi's Theorem and a

wonderful Hungarian mathematician called Imre Ruzsa came up after the talk and said that proof had three ideas. [laughs]

BH: [laughs]

TG: [laughs] Somehow it was a sort of a three idea proof, not just a one idea that…

BH: Triple proof.

TG: That…

BH: [laughs]

TG: That unlocked the argument. [laughs]

BH: [laughs] Nice. Just finally, 'cause I'd… I know I don't want to keep you all day. Well, I do want to keep you all day but I know I'm not allowed to. You talked about working on like AI and what machine learning and to do proofs, this is an area I find really interesting. Are computers gonna put mathematicians out of business? Is that realistic? Like you know, do you think it could get to the point where the computers are doing it?

TG: I think it will. [pause] Um… I don't know how long it will take. I think, although I could be wrong that… the current burst of activity in machine learning, while it's amazing for certain applications, is not going to be the thing that… puts mathematicians out of business without a certain amount of input which would not be easy input. [chuckles]

BH: Right?

TG: Into really understanding the processes that go on when mathematicians

do research. I… a sort of blind thing where you just train a network by showing it lots of proofs until it sort of suddenly starts spewing out proofs for itself.

BH: Hmm.

TG: I don't think that's very likely to be the answer. So that's not at all the way I've been… the type of thing I've been thinking… I've been really thinking about something that's always interested me as a mathematician anyway, which is what is it that's… I mean you've already asked about it. What is it that's going when we do research? How do we find these needles in haystacks and…

BH: Hmm.

TG: And I think that… that's a fascinating problem in itself and even if it doesn't lead to a computer program that puts us out of business it's a well worth thinking about.

BH: But you think it will? Like… so you're thinking it will just be a different kind of computer program. It will be a new… it will be like a paradigm change? It won't just be more and more power to the sort of pattern recognition and…?

TG: No, I think pattern recognition may well come in because there's no doubt that mathematicians do use pattern recognition. I think it'll be some kind of hybrid of… of… rule based logical thought, with quite a lot of non-determinism in it because… obviously we don't just apply some simple algorithm but that with the sort advances that there've been in machine learning. If you talk to a mathematician about how they found something out, it'll be a mixture of a very coherent narrative about, I was thinking about this and then I realized that such and such and that… thing that I've been trying couldn't work but then if you tweaked like this it… those sorts of things but there are other things where you ask a mathematician well… what made you think that that result was likely to be true? And they say, well I don't know. My just experience

kind of told me that the statement like that had a good chance of being true, I'm not quite sure why.

BH: My intuition.

TG: There's a certain intuition. Now I feel that the intuitive parts where… it's rather hard to explain… how you felt the way you felt as a mathematician. Those parts might well be amenable to machine learning because… somehow it's… that's seems to be what machine learning is good at. It's doing things that you do sort of instinctively and can't really explain. So I can't sort of explain to you how I know that a line drawing of a cat is of a cat.

BH: Yeah.

TG: I can to some extent maybe a little bit, I say well it's got a little pointy ear there but… I couldn't sort give you a systematic procedure for distinguishing between cat pictures and dog pictures.

BH: It's funny to me that you say that that's the thing the machines could do. 'Cause I think most people who are… defensive about machine learning, that's the thing they say the computers could never do. That's the one thing we've got is that… is that kind of instinct that a computer could never replace that aspect of my humanity. But you seem to think that's the thing the computers will do really well?

TG: Well that… I mean I'm just… I'm saying… I think they'll do the whole lot but I think that part, the sort of… instinctive judgements that we make, which aren't completely illogical but I mean they've got a whole… they can be justified to some extent at some point you get when you try to justify things you start running into difficulties. You say, well I just… you know, I've seen things a bit like that and that kind of I've seen things a bit like that and this feels like one of those… that feels that is a sort of pattern recognition that is going. So I think

machine learning would have a role to play… in those… so a big part of what you do when you do mathematics is what you might call probability judgements. If for example you… are trying to solve a problem and there's an intermediate statement that would imply an answer to the problem that you want to answer. It's not enough to say okay, well that would imply it, so I'm now gonna work flat out on the intermediate statement. There's another thing you need which is some kind of judgement of whether the intermediate statement is likely to be true. Whether it's likelier to be easier than the problem you started with. That sort of thing. Because otherwise you're risking spending a lot of time and not getting anything for it.

BH: Will I get a return on investment?

TG: Yeah, exactly, so that is a huge part of… efficient research is judging the return that you'll get on some kind of intellectual investment or time investment basically and so…

BH: Do you think that separates some of the better mathematicians from the inferior ones or less successful ones?

TG: I think it does, yes. I mean I think that sort of strategic… I do think… it really helps when you're doing research to step outside yourself and say, am I making the best use of my time by doing this? It's very tempting not to, it's very tempting to… and I've succumbed this temptation the whole time… just to sort of get interested in something, you're not quite sure why, it's maybe not most important thing in the world but it becomes the most important thing in the world for you [chuckles] for a while or another temptation that it's very easy to succumb to is just deciding… what you think the answer to a problem is and just more or less ignoring the possibility that it might go the other way because you just become emotionally attached and you want that problem to have a positive answer and not a counter example. As one example in my own career where I really was keen, had a really such a beautiful theorem and I never proved it and

eventually some other people found a counter example. [sighs]

BH: So you kind of dodge a bullet there?

TG: I sort of did, yes, if they hadn't found that counter example maybe I would've wasted a whole lot more time on it. But it wasn't a total waste actually. It rarely is, but… I do think time management is an important part and I've just said… just talked about examples where I've on the face of it not managed my time but actually I think also there's a sort of meta way of looking at things where not managing your time in a way that seems optimal can actually be good. So just getting a bit obsessed or emotionally attached with a problem can give you the motivation that you need in order to make progress with that problem whereas if you take on a more calculating approach to what was worth doing you might've thought, nah perhaps I'd better not.

BH: I mean I even in my career I've always sometimes being successful means doing what the other person's not willing to do, and that sometimes means those… things that appeal, like they won't be good return on investment.

TG: Yes, so now that's particularly, I mean that is another consideration I think when you're doing research. One of things you've… or… one of the very important judgements you have to make is not just, is this likely to be true? Is it likely to be easy to prove? But is this likely to be the way of solving the problem given that several clever people have thought about the problem and not solved it? If it's a fairly standard thing, if the idea… if there's no quirkiness to the idea… you have to have some plausible story as to why nobody solved a problem. Sometimes it's just, it's a problem you came up with yourself and it hasn't been something that other people have thought about but if it's something that other people have thought about, it actually helps this thing of narrowing down the search for what a proof could look like. It can be very helpful to say, it cannot be this because this is something that other people would've thought of if correct. This is a thing that cranks, mathematical cranks, don't understand. They come up

with some very simple proof of Fermat's Last Theorem and they don't realize that if it had looked like that, even slightly, and been correct, then it would've been discovered in the 17th century rather than… having to wait til the 21st century. But it applies also at a research level and I think sometimes an inexperienced research student may get excited about an approach that a more experienced mathematician will that, okay, it's good to think about that and understand why it doesn't work, but it's not gonna work because if it did work… [laughs]

BH: Yeah.

TG: …then it would've been done already.

BH: You don't find many like huge gold nuggets just still sitting on the ground when you go to gold…

TG: No, very occasionally happens and it's again one of those things that if it does it's slightly damaging…

BH: Yeah? [laughs]

TG: …in some ways because it encourages people to think that that's typical when it's not at all typical.

[gentle violin music]

BH: What are you doing at the moment? What's taking up like your time at the moment besides having me in your office and some posh dinner you've gotta go to tonight? Are you in a research mode at the moment or book mode or…?

TG: Definitely research. Although… at the moment I'm feeling a sort of pressure that… it comes in waves but a pressure of things that are basically done

but not yet properly written up.

BH: That sounds like a good problem to have? It sounds like the hard parts done.

TG: It is. It is, but it… uh… when you've got enough of them it can induce a sort of paralysis and one always wants rather than… particularly if they've been hanging around for a while, so it takes a while to get back into it enough to be able to work out how to change an old write up, that sort of thing.

BH: So these are proofs or solutions or things that you and your collaborators have nailed down and you've just gotta write it up now in the correct format and get it published… is that…?

TG: Yes, but sometimes when you've nailed it, it's… it can look really quite… horrible and complicated and…

BH: That's a really frustrating thought for me that there… that sitting in all these… on all these desks and in all these drawers around math departments around the world is like stuff that's gonna push the field forward and… no one knows about it just because it's hard to write.

TG: A few months ago I sat down and just… wrote a list of all the things that I've proved that I've not yet put out there…

BH: Tell me the Riemann Hypothesis is one of those.

TG: No, no, no, no, no, it's not that sort of thing.

BH: [laughs]

TG: There's… but if I were to write up everything I know, everything I've

done, what would there be and it was slightly depressing because there are things there that I quite like but I just kind of know in my heart of hearts they're not ever gonna get done because… there are other more important things that still need writing up. Particularly ones with research students where it matters a lot to them obviously.

BH: Hmm.

TG: When one's writing up it's usually it's a fairly thought provoking process and one always… it's always much more fun to be thinking about solving problems than just writing up things that one did sometime ago. Anyway so I'm in a phase like that, I need to do something about it and clear my backlog a bit and…

BH: Well, Professor, if that's not my cue to end this interview and leave…

TG: [laughs]

BH: …I don't know what is. [laughs] So, thank you.

TG: [laughs] Not at all.

[gentle string music fades in]

BH: [laughs]

[music continues]

BH: Did you ever grow a beard?

TG: Uh, no basically, actually this summer… just for fun I decided not to shave for a week or so.

BH: How'd it go?

TG: It didn't get…

BH: [laughs]

TG: A week wasn't enough for it to become…

BH: [laughs]

TG: …more than sort of designer stubble really.

BH: Right.

[music fades in]

TG: It was… it was interesting but I… it's not…

[music continues]

BH: It's not your look?

[music continues]

TG: I don't think so.

BH: [laughs]

TG: [laughs]

[music fades up]

BH: Well that's all for today, we'll include some links relevant to the show in the usual places. [music continues] Also as usual our thanks to the Mathematical Sciences Research Institute for supporting the podcast and thanks to the audio engineering company Meyer Sound, in Berkeley, California, for supporting this episode. [music continues] I'm Brady Haran, you've been listening to the Numberphile podcast and we'll see you again soon for another episode.

[music fades up and continues until fading out]