

Numberphile Podcast Transcript

Episode: Fermat's Last Theorem - with Ken Ribet

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Brady Haran [BH]: So Ken do I have to call you Mr. President?

Ken Ribet [KR]: Well, you can if you want.

BH: Do people call you Mr. President?

KR: Uh... sometimes jokingly.

BH: What honorific is used when you're like at meetings and that?

KR: I kind of live in Berkeley which is known for it's informality, so I'm very happy if people call me Ken, but I've been called Dr. Ribet and Mr. Ribet and Professor Ribet and Professor Ken and everything else.

BH: But like at AMS Board meetings when you're wearing your president hat is there something that they have to call you when they're like addressing the chair and that? Is there a proper honorific or...?

[gentle piano music fades in]

KR: Oh no, I have a gavel. And... basically we're all friends.

[music continues]

BH: Do you ever bang [banging noise] the gavel then? Have you ever had to like bang it or call something to order?

[music continues]

KR: Umm... basically not.

[music fades up]

BH: I'm Brady Haran and this is the Numberphile podcast, today our guest is Ken Ribet. He's a math professor at the University of California Berkeley. In the world of mathematics Ken's probably best known for his work on Fermat's Last Theorem. But hang on, I hear you ask, didn't Andrew Wiles prove Fermat's Last Theorem? [music continues] Well, yes, but this proof was a huge jigsaw puzzle with earlier contributions. And one of the penultimate puzzle pieces was put in place by Ken. [music continues] Today he's gonna tell us about it and what it was like to be in the actual room when Wiles proof was finally revealed. But first, in case you can't tell, I am a little bit fascinated by Ken's current role leading the American Mathematical Society, the AMS. [music continues]

[music fades out]

BH: How did you become president? Is it just like an informal thing among mathematicians or is it a competition? Is it like a campaign?

KR: Well, thirty years ago, forty years ago, I think each president would choose his successor. They were all males and at a certain point somebody to decide that it might be good to have an election. And I know the candidate who lost the first election, he was confident that he would just be elected without any question and his opponent actually campaigned by calling up some people and saying, hey, you know, I actually wanna do this so please vote for me. And he won. And there's a nominating committee that meets at the joint meetings and

also informally by email all the time. And they're considering candidates for offices like the president and the vice presidents and so on. When they choose someone as a potential candidate they approach that person and ask him or her whether or not person will run and in my case I was very flattered and I said absolutely and I ran in an election, I had one opponent who is very much like me in a lot of respects. He's from California, we have similar mathematical backgrounds, and I think both of us were viewed as very popular and very well known and there's no campaigning in the sense of debates or [chuckles] you know calling your opponent, you know, Lying Ted, or something.

BH: [laughs]

KR: It's still very collegial but I did make it known to my colleagues at Berkeley that I actually was going to be happy to serve. Very often if you see someone you know who's a candidate for something you say, well, are we gonna do that person a favor by voting for the opponent, but I actually wanted to do it. And I was... very happy that I won.

BH: Was it a landslide or was it a narrow one or...?

KR: Well this is confidential but it wasn't the sort of situation where we were having to look at hanging chads.

BH: Okay.

KR: But it was reasonably close.

BH: What is the role of the president? You've already referred to presiding over meetings of the council and the behind the scenes thing. What else do you have to do? Do you have to go and like cut ribbons?

KR: I do cut ribbons. There's that. Every... annual meeting, the President of

the American Math Society cuts a very large ribbon with an even larger pair of scissors to open the exhibits, the president presides over the joint prize session at the annual meeting. President comes to the congressional briefings in Washington DC, there are now two of them per year sponsored jointly by MSRI and the American Math Society.

BH: Well let's talk a bit about your background then, let's go back in time a bit, 'cause there's so many things I wanna ask you. One of the first things I wanna ask you is about your surname. Ribet. Where is that from?

KR: It's from Eastern Poland. It's from a town called Grajewo, which I've never visited. My grandfather was Ribetzky, and when he came to the United States in the early part of the 20<sup>th</sup> century, it was the fashion at Ellis Island to make everyone American. As American as possible, and so I became Ribet, which has become very useful to me in France and many of my colleagues they would phone up restaurants to make reservations and try to explain their last names and if the last name didn't sound French, there'd be a lot of hesitation, trying to get the name right and so on.

BH: So people do think it's French sometimes. Ribet [RE-BAY] or something or...?

KR: Yeah, well there are many many French Ribet [RE-BAY] and in fact on Facebook you can see there're big clusters of them and there are people who've contacted me saying, my ancestors came from France through Cuba, were your ancestors in Cuba? No,

BH: Poland. There was the Ellis Island Corleone moment for your family where the name changes in just a few seconds.

KR: Right.

BH: Where were you born? In America?

KR: So I was born in Brooklyn, New York. My parents were both born in New York City, and when I was three years old we moved to a place called Rockaway which is a part of Queens, but it's a pretty remote part of Queens. Like they don't even have a Starbucks nearby. [chuckles] To show you how remote it is. And recently it's become very Brooklynized because it has surfing and so all the hipsters come from Brooklyn now on the weekends and many of them have actually moved to Rockaway. So for example the rock singer Patti Smith bought a house.

BH: Were your parents... mathematical, academic type people?

KR: My father was a CPA, an accountant and when I was a little boy he would always be adding numbers. Either by hand or with a small adding machine and I was fascinated by numbers and I would ask my dad to give me, you know, long addition problem and I think that's what hooked me in.

BH: This was the start was it?

KR: Yeah.

BH: At school, were you gifted at mathematics? Was that easy for you?

KR: It was my best subject, yeah. So I was a pretty good student. Terrible athlete, complete nerd, but I was good at school in kind of all subjects and math was the easiest.

BH: And was that the path you wanted? Like if I was to talk to the teenage Ken Ribet would he have said, ah I wanna be a mathematician one day?

KR: I don't think I knew that there was such a thing called a mathematician.

I've told the story many times, but it started when I was a freshman at Brown, I went to my first mathematics course. It was a course in abstract linear algebra. And I loved the material and I was really taken with the professor and I, you know, looked at this guy, his name was Frank Stewart, and I said, I wanna be like that. This is what I wanna do. I had still no idea what it actually meant, you know, I didn't know what Professors did, how they spent their time, you know, there's this thing called research and committee work and reviewing and traveling. I had no idea what the game was, but I was very taken both with the content, with the mathematics, and also with this kind of freedom from the constraints of business.

BH: What were you at Brown doing?

KR: Well, so, I showed up as a freshman with some vague intent to try math and applied math and chemistry, I took one applied math course, it was a year long course, I didn't like it very much, I never took a chemistry course so I went straight for math. I did math and broadcast radio. I was on the campus radio station.

BH: Oh, well I'll be watching your microphone technique with care then. You have to excuse my ignorance about the US college system then. You had been accepted and turned up on Brown to start your university career and it still wasn't known what you were going to be studying?

KR: That's correct. In the US, when people come to college they are not really expected to declare their major until their third year and so there's this idea that they can experiment and try different things. In fact it's very common. I mean if you look at what people are majoring in on, say, the Berkeley campus. Well, they're majoring in things like social relations in anthropology, you know, theater, it's very rare to take a high school student and say whaddya wanna major in? And people will say anthropology. You know people talk about their core subjects. Maybe they'll talk about computer science, but they won't think,

well I wanna be in geography.

BH: So Ken if I was America's most gifted mathematics student at high school and Berkeley wanted me for this reason and offered me a place, I could then say, ah thanks for my place at Berkeley but I'm not really interested in mathematics, I happen to be good at it but I wanna be an actor. And I could just do theater and never study a jot of mathematics?

KR: Absolutely.

BH: Alright.

KR: Okay, I mean, the admissions boards, they choose people to try to sculpt the profile of their entering class and they have an idea of who's coming in and they could be completely mistaken.

BH: This is new information to me. You did follow this mathematical path and then you decided pretty early on, ah I wanna be like the role model professor and you did that eventually, did you?

KR: I did. So I'm the rare example of somebody who set out on a path and just kept walking straight. I mean most people, you know, Yogi Berra said if you come to a fork in the road take it. I mean most people end up doing things that are completely unpredictable. And my thing, you know, I just kept going forward and there was no obstacle and I liked it more and more and here I am.

BH: And then you started having a specialization in mathematics? What was the field of math that ensnared you?

KR: So it's a blend of number theory, which is the study of whole numbers and geometry, which, also pretty well understood what that is. So there is a kind of number theory slash algebraic geometry subject which has been coined

Arithmetical Algebraic Geometry. So that was my subject and I came to it because I had some professors at Brown who were kind of very very forceful and they said you're very good in math, read this paper. And these two people. Ken Ireland and Michael Rosen and they were fantastic. You know, they were doing interesting stuff, they were interested in things and they kind of kept pushing me in different directions. They told me where to go to graduate school, who to work with, what subjects were interesting. And it just meshed well with me.

BH: So was it an aptitude or was it peer pressure?

KR: Well so I think I was lucky in that I was pushed into a subject that I'm actually good at. I believe that people have very uneven aptitudes even within a subject like mathematics. You know, I may not have been very good at geometry, or probability or some other subject. I probably would've been okay because, you know, I was... kind of good and all around. But by luck I was pushed into a subject that really spoke to me and I was able to speak back after a while.

BH: At this point is your ambition to be a teacher, like, you know, a professor in front of a class and inspiring mathematicians or are you wanting to be like a Gauss and an Euler and, you know, the person who's making the discoveries and having the mathematical fame?

KR: I think the mix is the best thing because, if you do several things and on a given day you're not so good at one of them well you turn to the other whereas if you're just focused on one thing, like for example suppose you have a sabbatical year and you're at a math institute and you know that the entire year you're going to be doing research, well at the end of a day if your pad of paper is still blank... you feel terrible. You haven't done anything. Whereas if you start thinking about something and you kind of keep going in a circle and you've just come back to what you've done before, you say, well, you know, maybe it's time to make up this week's homework.



[gentle chime music]

BH: We're gonna come to Fermat's Last Theorem shortly, in which you play obviously a huge role, but at this point when you're just starting out in this field and you're like, you know, you're one of the new kids on the block, what was the holy grail problem of your topic? What was the thing that you would be lying in bed thinking, gosh if I could be the person to solve that, that would be amazing?

KR: Well there are a lot of long term goals for the subject, one of them curiously is the theorem that was proved by Andrew Wiles with the help with Richard Taylor and Brian Conrad and so on. The Modularity of Elliptic Curves, that was a very well known conjecture that I first heard about as a first year graduate student at the time it was called Weil's Conjecture after André Weil who wrote a paper related to it in 1968. This was kind of a holy grail but not in the sense that, you know, everyone's trying to do it, it just basically colored the landscape.

BH: But at that point was that not connected to Fermat's Last Theorem?

KR: Absolutely not. It had nothing to do with Fermat's Last Theorem, the thing that I studied that was my thesis and a lot of my subsequent work, one topic is Galois representations and modular forms which came into the proof of Fermat's Last Theorem. So when I was a graduate student there were countless courses and seminars on subjects like Iwasawa theory, Galois representations, modular forms, these are all things that came together, you know, twenty years later. Twenty-five years later, in the proof of Fermat's Last Theorem.

BH: Were these kind of like Riemann Hypothesis type things where there was already mathematics built on assumptions and you need to prove it for that to hold, or was it more just like something in the distance like a finish line?

KR: Oh it's a mix. I mean so certainly over time people became more and

more confident that elliptic curves would end up being modular. They thought, you know, it was true but something we couldn't prove, so they began kind of deriving consequences of it. So that if you derive a consequence of an unproved conjecture you have a theorem, you can state it in different ways. You can say assume all elliptic curves are modular then such and such is true. Another way to spin it, you can say that, all elliptic curves that happen to be modular have the following properties.

[gentle piano music]

BH: Perhaps at this point we should deal with Fermat's Last Theorem actually is. Now there are a few Numberphile videos about it. [music continues] You can go and check them out, I'll put links down in the show notes. But let's give it a go in audio form.

[music continues]

BH: Now imagine in your head a pretty simple equation, and quite a famous one. A squared plus B squared equals C squared. [music continues] I'm sure you're familiar with this one, you probably remember it from Pythagoras at school and the right angled triangles and the question is, what three positive integers, so whole numbers, can you put in the place of A, B, and C, so that the equation works? A squared plus B squared equals C squared. [music continues] And, well, in this case, there are lots of possible answers, in fact there are an infinite number of answers. For example, three, four, and five. Three squared plus four squared equals five squared. You could use five, twelve, and thirteen. You could use nine, forty, and forty-one. You could use thirty-three, fifty-six and sixty-five. There are all sorts of ways to crack this nut. [music continues] These are called Pythagorean Triples. But let's up the ante a little bit. Let's not square those numbers let's cube them. Raise them to a power of three so now we have A cubed plus B cubed equals C cubed. Hmm. Are there three positive integers we can put in there that will make this equation work? You can give it a go if you

want but don't spend too long on it. [laughs] What about if we raise it to the fourth power? [music continues]  $A$  to the four plus  $B$  to the four equals  $C$  to the four? Or any number, any  $N$ .  $A$  to the  $N$ th power plus  $B$  to the  $N$ th power equals  $C$  to the  $N$ th power. Are there numbers we can put in their place, positive integers, that will make this equation work? [music continues] Now back in 1637, a famous guy called Pierre De Fermat said, no. There's not. And I've got a proof. He wrote in the margin of a book, he just hand scribbled, I've got a proof that you can't do it with anything, except of course the squares. [music continues] But he said, I haven't got room here in the margin to write the proof, I think, you know, I'll come back to it at a later date. He never did, he never wrote the proof, no one ever saw it, and this became as Fermat's Last Theorem. Was there are a proof that there were no positive integers that you could put in the places of  $A$ ,  $B$ , and  $C$  with all these higher powers that would work? Was this impossible? [music continues] For hundreds of years, no one knew, they never found any, but there was no like rigorous proof. Now that finally changed when an Englishman named Andrew Wiles gave a famous lecture and revealed a proof. And suffice to say, you wouldn't be writing this proof in the margin of a book. [music continues] It was really really complicated. Really advanced cutting edge new mathematics. So there's Fermat's Last Theorem, I hope you have some idea of what it is in your head. We're gonna get back to it in a minute, but let's get back to our discussion with Ken and find out where he's at in the years leading up to this big moment. [music continues and fades out]

KR: Well I personally was doing pretty well, although you know I had a deep sense of insecurity, you know, was this worth anything. I wrote a thesis at Harvard that I was kind of vaguely proud of and I wondered whether anyone'll offer me a job.

BH: What was the title of your thesis?

KR: Galois Representations Attached to Abelian Varieties with Many Real Multiplications.

BH: Catchy.

KR: Okay, so that was the title and in fact my thesis, you know, I wondered whether it would even be published. It kind of sat on my desk for a year and a half and finally a mathematician named Serge Lang grabbed it and he said I'm sending it to the American Journal and I'm gonna tell the editor in chief to publish it. And he did that, and nothing happened for about two years. And finally I kind of inquired, you know, whatever happened to this and they asked and finally it just got published. You know, I was wondering if anyone would ever read it, but it turns out... it has lots of citations and people think it's a good thing for background. I was waiting, you know, to hear whether any place would offer me a job and before I expected job offers to come I got a phone call from Princeton University asking me to come and be a basically a postdoc. So I was there for... it's hard to count to the years because the third year that I was supposedly in Princeton, I got a fellowship to study in Paris. So that was my first year in Paris, and I actually taught at Princeton for a total of three years with this year in Paris kind of sandwiched between the second and third year.

BH: You got to take that Ribet [RE-BAY] surname out for a test drive?

KR: Yeah, in fact one of the first things I did was to take the Metro to the Impasse Ribet, which is a little alleyway, and I've certainly have photos of myself standing in front of the Impasse Ribet.

BH: Excellent. And then what happened after that three years? Where were you next?

KR: In the last year at Princeton I got a phone call from Berkeley saying, you know, how would you like to come out and join our faculty? So this was completely unexpected. I hadn't applied to Berkeley but secretly, you know, when I thought about universities where I might go, Berkeley was probably at

the top of the list, 'cause it was one of the top three universities in terms of mathematics, the other two being Harvard and Princeton, and I had visited Berkeley once for a week on my way to a conference, I just thought it was paradise. So I tried my best to hide my enthusiasm when they phoned me up. I ultimately accepted an offer of associate professorship at Berkeley and promptly went to Paris again for a year. And so I first came to Berkeley kind of a year and a half after the initial phone call.

BH: What was it about Berkeley, 'cause the Ken I know, you're Berkeley through and through, like you're such a Berkeley guy to me, but like I guess then you were a real East Coast guy, so what was it about Berkeley that appealed to you?

KR: Well it was kind of the light and kind of joy, the informality and Telegraph Avenue is full of people wearing tie-dye shirts and it was kind of like the Sixties was still going on.

BH: But you're an East Coast, wouldn't you look at that, like shun that? Weren't you... this Brooklyn guy who would think, oh those hippies over there?

KR: Well, you know, I'm thinking of someone I know who works in San Diego who... applied for a job and in his interview he said, I'm a Californian trapped in the body of an Englishman. [chuckles] So maybe I was a Californian [laughs] trapped in the body of an East Coast person.

BH: Okay that makes sense. Let's jump forward then to Fermat's Last Theorem. What did you do? How did you suddenly find yourself in this spotlight? This is before Wiles, before the proof.

KR: This is before Wiles, so this is in the early to mid 1980s, there was this guy Gerhart Frey who keeps coming up in all the discussions of it and he was the first person that I knew of to try to think about the links between elliptic curves and

solutions to Fermat's Last Theorem. So at the time there was a lot of study about elliptic curves with various properties and if you had an elliptic curve with various properties... you could derive from that elliptic curve the solution to some equation, and there was a movement, one of the mathematicians was a Russian mathematician Demyanenko, where you tried to rule out certain kinds of elliptic curves by deriving from the existence of the elliptic curve a solution to an equation that didn't have solutions. So that was the subject and it was going back and forth between elliptic curves and solutions and Frey somehow stumbled on this idea that if you had a solution to Fermat's Last Theorem, this would give you an elliptic curve with extremely problematic properties. [pause] And he kind of went around telling people about this and he came to Berkeley and spoke to me and gave a talk and I was very unconvinced. I kind of really didn't believe it. This was around 1982 or 1983 and over the next year or two he came to this idea that if elliptic curves were known to be modular it should be possible to prove Fermat's Last Theorem. And he wrote a manuscript that was like two or three pages with an outline of the assertion that the modularity of elliptic curves implies Fermat's Last Theorem and he was well aware of the fact that his outline... was not at all complete. But he was hoping that people who were experts in the theory of modular curves, like me I guess, would fill in all the blanks and kind of make the thing okay. I was still completely unconvinced, I didn't kind of know that he was right, but he gave lectures in Europe and a lot of European mathematicians in 1984 or '85, that academic year, started thinking about his lectures and his little manuscript and thinking about ways to... justify this idea that the modularity of elliptic curves could imply Fermat's Last Theorem. And one of the single things that happened was that Jean-Pierre Serre wrote down a longer manuscript, it was a letter to Jean-Francois Mestre, French mathematician, explaining that if you could only prove what Serre thought was a tiny little assertion, he called it Epsilon, meaning a small number, then you could actually justify what Frey had started to do.

BH: So this was like a little screw, or just a little piece of the puzzle but it was really really important to hold it all together?

KR: Yeah. So in other words Serre's letter basically said, Frey looks like he's right, you just have to prove this one extra thing. And this was in the summer of 1985 and I got a copy of the letter, we were all together at some conference in Northern California and I started thinking about this Epsilon and it was really right in the center of things that I understood.

BH: Were you thinking about it, Ken, because it was like an itch in your brain and a curiosity or were you thinking greatness awaits if this can be solved?

KR: Well, I thought that the connection with Fermat was kind of the icing on the cake, but I'm not one of these people who say, you know, I've kind of went to the library when I was a ten year old and read about Fermat's Last Theorem and it was my great goal, it was a challenge, it was a mathematical challenge, it was right in my subject, it was something that I thought I could do, and I was responding to that challenge. The connection with Fermat made it, you know, even more important to think about.

BH: Like any mathematician, you knew of Fermat's Last Theorem, it was just like a famous thing but I'm not gonna ask you to explain it and the proof and the mathematics 'cause I wouldn't understand it and you can't do it in a podcast. But how did you do? Where did you do? Did you do it lying in the bath? Were you walking along the bay? Did you just sit in your office with a notepad and a pencil? Where did you crack this nut?

KR: Oh, so I thought about it off and on for a year. This was the academic year 1985, '86, and I... you know was teaching Calculus and whenever I had a spare moment I thought a little bit about this problem and asked myself whether I had any special insight into that other people didn't have and at the end of the teaching, so this would be in May 1986 I started thinking about the problem more seriously. I was on my way to Europe, so the first thing that I did was I went to Harvard for basically a week, so I had an office that I was able to use and I was

also staying with my sister in Wellesley. I remember sitting in her kind of breakfast table in Wellesley thinking about this thing, and realizing that I knew something that I hadn't realized before that I knew. That I had the beginning of some complicated argument that seemed to do a special case of it, so I got very excited, and then I flew to Europe, I was in Paris for a while and then I showed up at the Max Planck Institute in Bonn and in all these places I had my pad of paper and I would write the thing and by the time I got to Bonn, so after my arrival, this is probably around June 1<sup>st</sup> 1986, I realized that I could do what I thought was only a special case of the Epsilon Conjecture. So the Epsilon Conjecture, you try to understand something mathematically, you say, well what's the first possible situation where you have to prove something. The simplest possible situation. So I kind of looked at the theorem in that case, and this was really right up my alley, it was this kind of thing that I studied many times before, in fact, I had studied Conjecture Epsilon and the special case kind of ten or fifteen years before and realized that I didn't know how to do it and wondered whether it was true, but here I was thinking about the problem again and I realized that I could actually do something. So this was kind of very exciting to me and I kind of wrote the thing over and over again on different sheets of paper and I convinced myself that it was actually true and I started talking to people about it in Bonn and I might have given a lecture or two in Bonn and I also wrote a letter, a physical letter on paper, to Barry Mazer at Harvard and I mailed him the letter and I didn't hear back. [laughs] And then there's this famous story with the cappuccino in August of 1986 in Berkeley. There was the International Congress of Mathematicians, that happened to be in Berkeley, Barry Mazer was there and I said, hey, you know, I sent you this letter and I'm trying to work on the general case and you didn't respond to the letter and Barry looked at me and he said, well of course, you know, you have done the general case, you just have to do something slightly different. So he thought he was just, you know, sprinkling some powder on it and telling me something that I should've realized long ago. So we went and had a coffee, at some cafe at the edge of campus, it's now called Cafe Strada, and we sat there and as soon as he explained the thing in one or two sentences, what I had to do extra, I realized



that there was gonna be no problem, that the thing was going to work.

BH: Why did Barry not reply to your letter, to tell you that? Or did he think you already knew?

KR: Oh, I don't know. I mean maybe he was thinking about other things or he thought I already knew and I guess his attitude was, well Ken'll figure it out by himself.

BH: [laughs] What do you say to Barry about that now? Do you kick him and say why didn't you tell me a year earlier?

KR: Well I don't think we've discussed this subject recently.

BH: So when you publish this proof and it had all been set in stone, this sort of bridge or pathway to proving Fermat's Last Theorem had been opened, but not yet passed through. Was that a big deal? Like were you a celebrity for a week? Was this, oh this is amazing or because the last piece, because the step onto Fermat hadn't been made yet, was it still just an obscure mathematical paper in the scheme of things? Or was this like making waves?

KR: No, it'd made a lot of waves. Not to the extent that Fermat's Last Theorem itself made waves but it made waves within mathematics. So I was approached by journalist who wanted me to kind of talk about the proof for various popular and scientific magazines. Everyone knew about it in mathematics, that this thing had happened and there was also a certain amount of skepticism, is this proof gonna work out? You know, 'cause I didn't have a complete manuscript, it hadn't been accepted for publication. I gave lectures, I explained all these things. And a lot of the proof was so convoluted to people that they thought it couldn't possibly be correct. At least some people felt that way.

[gentle chime music]

BH: You and others have helped open this door and shown this path, does this create like a mad scramble? Is there now a race? Is there a frenzy, now Fermat's available? And are you in this race?

KR: I thought that the assertion that needed to be proved, the modularity of elliptic curves, was completely beyond the techniques at that time. So I thought what I had done was to convince people that Fermat's Last Theorem... was true morally because people believed in the modularity of elliptic curves but I had no expectation that anyone, you know, seven years later, would claim to prove that elliptic curves are modular.

BH: A bit like the Riemann Hypothesis, everyone believes it's true but no one seems to be able to prove it?

KR: Right, so I thought this was a completely inaccessible problem.

BH: Did people start trying though? Did it create this new buzz? This new field of endeavor?

KR: Well... not that I was aware of. Now it turns out of course, Wiles went to his attic and worked on this thing quietly but apparently other people were having kind of similar ideas, you know, maybe there's a way to do this.

BH: Did you know Andrew Wiles? This guy in England?

KR: Oh yeah. So I met him the first year... that I was in Paris... let's think about this. This was 1975, '76. So I was in Paris for the year and Wiles' advisor, John Coates, would invite me to Cambridge basically he said, you know, whenever you're tired of Paris and wanna spend a week in Cambridge, just come over, and we have a room for you, you give a lecture, we can pay some travel

expensive, we'll invited you to High Table, so I came to Cambridge, you know, maybe four or five times. Maybe even more during that year and Wiles was Coates' student. And in fact the thing that I had done by the end of the year was something that also created a lot of interest in mathematics. I proved what's now viewed as the converse to a Theorem of Herbrand. Jacques Herbrand. And I was lecturing on this in Cambridge and Wiles got very excited and one of his first papers was to take what I did and go beyond that. And then his most famous paper after that was a joint paper with Barry Mazur where they proved something called the Main Theorem of Iwasawa Theory and they did that by using techniques that were derived from mine.

BH: So, part of the mythology of Andrew Wiles now is that he was obsessed with Fermat's Last Theorem from boyhood. Did you know this? Would he always, every time you met him, would he talk to you about, oh how do you think we can solve Fermat's Last Theorem or was this... did he keep this secret or like did you know this was one of the people in the world?

KR: I had no knowledge of it. I mean, I take him at his word, and in fact what he says is that he was obsessed with Fermat and the... adults in the room told him, you know, don't think about that, here are some fruitful problems that you can think about, and he took their advice. But he did, at least by self report, have this special obsession.

BH: So anyway, he does go up in the attic, well eventually he does crack this nut. How did you find out about this? You were kind of almost part of this too, not part of the work but you were very Johnny on the spot when this all emerged into light, weren't you?

KR: Well okay, so in June 1993 there was a conference in Cambridge and I was invited, I was one of the speakers. And at a typical conference you have, you know, a number of speakers, maybe four or five, six per day. And no one speaks more than once. And Wiles had told the organizers that he had something very

special that he wanted to present and it would require three separate lectures on three consecutive days and he was booked for this and after the first and second lecture there was widespread speculation that he was gonna prove the modularity of a very wide class of elliptic curves and this would be enough to prove Fermat's Last Theorem.

BH: But this wasn't like the title of his talk, it was, cloaked in some secrecy was it?

KR: Well, the title of his talk... I mean in fact I joke that both in my article and in Wiles' article, Fermat's equation appears in the introduction and never after that. I mean the both articles are about the technicalities of modular forms, modular curves, Galois representations, these are the tools that we'd all been working with and Wiles' lectures had a title like that.

BH: But it becomes clear that day three might be the big moment, and you cottoned on to this, didn't you?

KR: Yeah, in fact, someone kind of whispered to me that this was gonna happen after the second day. So it wasn't a surprise to me because I actually had some knowledge, but even if I hadn't, you know, I would've expected like that and there was a huge audience for his lecture. A lot of people had cameras, a lot of people brought their friends who were not in the subject and they said, you know, come to this lecture something historic is gonna happen.

BH: What was it like being in that room?

KR: It was joyful. It was electrifying. It was kind of a celebration. You know when I was a graduate student, my relatives would say to me, well what are you doing this for? What kind of subject is this. And I would say... well you know, basically what were trying to do ultimately is solve equations, 'cause that's something people could understand, and you know, it was wonderfully

validating after all these years that we were actually solving an equation.

[pause]

BH: You took a camera for which I am forever grateful because I think you got the photo.

KR: I may have. I certainly had a film camera there and I took a number of photos and then there were some group photos that I appear in that were taken by other mathematicians who were there.

BH: There's one of him like at the blackboard.

KR: I think I took a photo like that, yeah.

BH: Yeah, he's smiling and it's like. He doesn't seem like the world's smiliest man and he's got this like glow about him.

KR: He did, he couldn't suppress his pride.

BH: And straight afterwards this was like all this buzz and attention and you kind of ended up having to be a bit of a spokesman, didn't you?

KR: Yeah... first of all, the staff at the Newton Institute knew in advance that something was gonna happen and they chilled cases of sparkling wine. Was actually Napa Valley sparkling wine.

BH: Of course.

KR: That was served after his lecture. And there was a lot of buzz and as soon as the lecture ended someone came up to Andrew and said, Gina Kolata from the New York Times is on the phone, would you speak to her. And Andrew turned

to me and he said, Ken, you talk to her. [pause] So I went into an office, it was actually my office and I sat underneath my desk [laughs] in order to have some sound proofing 'cause there was a lot of chatter in the hallways. And I spoke to her, you know, for maybe a half hour. And I outlined the whole thing and the next morning our names were on the front page of the New York Times, which is kind of remarkable, so like my parents, you know, got the New York Times delivered to their driveway and they open it up and there's their son on the very front page and... there was this possibly unfortunate term that she used. She kind of referred to me as Andrew's spokesman. So it was true that he asked me to speak for him in that context... but then somehow it became known in the press that I was the official spokesman for Andrew Wiles, so I was kind of like his press secretary or something.

BH: [laughs]

KR: And I got hundreds of inquiries from all sorts of publications and radio shows and TV shows and my attitude was I would speak to everyone and they asked me the same questions over and over again. You know, did Andrew Wiles prove Fermat's Last Theorem? And I basically said, yes. And occasionally I would mention that I had done preliminary work but that rarely made the write ups.

BH: No. [laughs]

KR: It was all about Andrew Wiles.

BH: There's a whole Numberphile video about that though, Ken. [laughs]

KR: Okay.

BH: [laughs]

KR: It was an interesting situation to kind of deal with the popular press. I'd never had this happen in my life before and I found that I was fairly good at it. I could explain things to people who didn't know anything about mathematics and I thought it was a good use of my time.

BH: The other famous part of this story of course is that there Wiles' proof was then found to have a flaw. What was that like for you? Did that hang you out to dry?

KR: It really did because first of all when Andrew first heard about the flaw in July of 1993, I guess he didn't believe that it was all that serious. He thought it would just need a patching up. So, he must've been aware that I was out there, you know, beating his drum, but he never told me. So it took months before I found out how serious the problem was and I had been... kind of out there saying, you know, Andrew Wiles had proved Fermat's Last Theorem. So, I'm glad that it worked out.

BH: Right. [laughs] So just to patch up that story, Wiles worked with another mathematician, Taylor, and they did fix the problem. They brought it back from the brink and it all held?

KR: Yeah, so Taylor and Wiles huddled together and they thought about various things and finally they, or Andrew or Richard... found the missing piece and they published that as a joint paper.

BH: You talked about when you were working on Epsilon, you were giving lectures and you were writing letters. You wrote letters to Barry Mazer. It was this open book, wasn't it? You were showing everyone how you were progressing and asking for help.

KR: I was.

BH: Andrew Wiles famously sort of didn't do that. He kept it very secret and then he revealed everything at once in like a big grand reveal. Does that confuse you or does it delight you? How do you feel about these two different ways of doing mathematics?

KR: [pause] Well I... certainly like my model better, but Andrew had a very good excuse which is that Fermat was such a kind of sought after thing that if he revealed that he was working on the problem, people would've rushed in. And it was very important to him to be the person to prove Fermat's Last Theorem.

BH: I'd keep it secret if I was close, I think but...

KR: Yeah.

BH: ...then again, I'm not a mathematician. So you've played a great role in this amazing thing. Probably one of the greatest things that's happened in mathematics in the last, I dunno, hundred years or so, certainly? Where do you go from there? What do you do next? Even you, who like, you know, was a side player in it, it's still like a big deal. It must be one of your crowning achievements? What do you do next?

KR: Well there is an encore problem. There's no question about that. And what I've done, if I look off to the side and say, you know, what has Ken Ribet done? Well, I've done a surprising amount of outreach and professional service. And I think that's something that I'm pretty good at. I've done more research, I've had graduate students, I've solved problems, I've proved theorems but... I haven't done anything as great as the contribution to Fermat's Last Theorem. Realistically I'm not looking for that, so I'm looking for some balance where I continue to contribute to research and also now I'm a pretty well known and well regarded sort of senior mathematician, so I do things at the National Academy of Sciences, I'm also kind of the Chair of the Class of Math and Physical Sciences at the moment. I work with the American Math Society. So I feel that I'm still



contributing in different ways.

BH: Do you think you can't contribute in those other ways anymore? There's this great cliché or this thing that's said about, particularly in mathematics, that you know, your prime is leading up to about forty and after that you haven't quite got it like you used to have it? Do you buy that, do you believe that?

KR: Well there's... you kind of maybe lose some speed but you have lots of wisdom and you can recognize situations that some people may not.

[gentle chimes]

BH: Let me talk to you about photography and photos. 'Cause you seem to love photos.

KR: I do love photos.

BH: And you're always taking them. We've taken pictures even today there was like a fan here in the room that captured your eye and you wanted to take a photo of it. What is it about you and photography and photos? It's just like a hobby?

KR: It is a hobby, there's no question about that, so when I first came to Berkeley, I had no camera, and I was as I said fascinated by the light and all the scenes in Berkeley so I bought my first camera.

BH: What was it?

KR: It was an Olympus OM-1. So this is a film camera, and it had no flash, and I went around and I took photos and I would get them developed in photography stores. So I'd come back, you know, with color photos, and then someone said to me, well you really like photography, why don't you do black

and white photography and go in a darkroom. And there is a darkroom on campus and the dark room is beautiful, had lots of equipment. There were courses you could take with accomplished Bay Area photographers. So I did all of that and I spent a lot of time in the darkroom and it became the thing that I would do at the end of a day of mathematics, you know, at five o'clock I would go in the dark room and I'd kind of emerge three hours later with lots of prints and lots of negatives and start thinking about dinner and with digital photography it became a lot easier, you can just snap pictures and post them in various places so it's kind of a sideline.

BH: Anyone who's been on your website, in addition to all the mathematical content on there. There's all sorts of little bits of trivia and interesting things and one of the sections I can't stop looking at is the haircut section. Tell me about that.

KR: Oh... well, so many years ago I started taking haircuts with one particular stylist who is no longer coming to Berkeley and we together hatched this idea of taking photos of the two of us at the end of every haircut. I kind of believe in the power of a sequence of photos. Even if one photo doesn't reveal all that much, having a long sequence of similar photos again and again shows you kind of the arc of time and there are very details that appear and disappear. People walking past in the back, so I really like that, and I took a photo of every haircut and then when my stylist decided that it was no longer worth her time to drive to Berkeley from San Jose, the photos ended. And this was quite a few years ago.

BH: The other thing, I mean, I follow you on Facebook, and the other thing I find interesting is, you have these really regular lunch meetings. Explain what they are.

KR: When I was a student, both at Brown and at Harvard, there was some tradition that faculty members could have lunch and other meals if they wanted to with students and I found it really valuable to me to meet all sorts of people who would come and sit around with students. I remember for example meeting

Yo-Yo Ma when he was freshman, he was at one of the tables and there were lots of public figures who would come in and sit with us and I thought that was really wonderful and when I came to Princeton I asked whether or not there was such a program and there was so I became a faculty fellow at various dining halls and when I came to Berkeley there was no such thing and I kind of wondered for a long time whether such a thing might crop up. And when I started teaching very large courses... that had formerly been small courses in the math department at one point we began teaching third year courses in large rooms, whereas before they'd only been taught in groups of twenty-five or thirty students. I thought you know, this might be a way to get to know the students and I would organize lunches in the dining halls. And I pitched this to the College of Letters and Sciences and they were all on board. They gave me a card so that I could get free lunches and I would see with the students and I found it was really valuable and they liked it and then somehow this morphed about five years ago into organizing breakfasts and lunches at the faculty club. And the faculty club likes it, they get lots of business, the students like being in the faculty club and I tell them they can come back anytime they want, they don't need a faculty member, all they need is a credit card or a wallet and some of the students have taken me up on the suggestion and I think it's a nice way to get to meet students informally and find out a little about their lives.

BH: So you normally put like a post on Facebook, don't you, and you basically say this week I'm gonna be here at this time, come along if you want, and then this handful of students will come and you'll just hang out over a meal?

KR: That's right.

BH: What do they normally want? Is there a pattern to what happens in the conversation or...?

KR: It depends on the size of the group, so a small group they'll ask me about myself and they'll be a little bit uncomfortable, you know, it's kind of

intimidating to be with a faculty member and they're addressing questions to me like, you know, how did I come to be a mathematician, what do I do in my spare time, what is it like... how long have you been at Berkeley. So those sometimes if I don't know the students, can be, a little bit rough, but then there are bigger events where there'll be you know, ten, twelve students and then what happens is they just talk to each other. And I'm there kind of almost like a fly on the wall. I mean some of them will talk to me but everybody's much more relaxed and I find those to be the most successful.

BH: And then coming back to your passion for photography, it seems like at the end of almost every one of these there's like a group photo.

KR: There's a staff member at the faculty club, Jerry Fowler, who is a fantastic photographer, and he takes photos of the group. And he really gets the best expressions out of people. I think people warm up to him immediately.

BH: So if you go like on your Facebook, almost every week there'll be this... it's almost like the exact same picture with a different cast of characters each time.

KR: Right, you see the seasons changing, the colors of the leaves, some of the photos everyone's wearing an overcoat, some of the photos people are in summer clothes.

BH: You do seem to enjoy teaching more than a lot of other academics I know. Sometimes academics seem to treat it like it's their chore. It's the pound of flesh they have to pay to be allowed to be researchers but you seem to really delight in it.

KR: Well, it's a kind of communication. It's a different skill. It's a different activity. It's not explaining technical math to people who are committed to mathematics. And I think everyone... on the Berkeley campus because it's such a

large public university, is committed to that kind of teaching, because someone who comes here and realizes that... there's that kind of teaching almost every semester and doesn't like it, will go elsewhere.

BH: And what's next for you then, as we come to the end of our chat? What's your next big thing? I guess the next part of your presidency is a big part of your life at the moment?

KR: As my presidency is coming toward an end, it ends on February 1<sup>st</sup> 2019.

BH: And you're term limited?

KR: Well there's never been a situation where someone has done it twice. My successor, Jill Pipher is a mathematician at Brown. After her there are two candidates who are now recruited for the next election. One of them will be Jill's successor. On February 1<sup>st</sup> 2019 I will become the official Immediate Past President. So this is an actual title of the AMS. And for example as Immediate Past President I will be the representative of the American Math Society to a joint meeting with the Vietnamese Math Society next June.

BH: You're taking on almost like a deputy or a vice presidency type role when you're the Immediate Past or...?

KR: Yeah, so in every moment there are two presidents in the AMS. There's the President and then there's either the President Elect, who shadows the President, or there Immediate Past President, who helps out as needed. And after my one year as Immediate Past President, the next person who will've been elected will sign as President Elect.

BH: And they you got any plans we should know about? Any... you're not secretly working in an attic on another proof, are you?

KR: Uhh... I'll... I'll stay mum as to my... my plans.

[gentle chimes echo]

BH: [gentle music fades in] Thanks for listening. This episode was recorded in a small office overlooking the San Francisco Bay at the Mathematical Science Research Institute, which is in Berkeley, California. [music continues] I'm sorry you couldn't enjoy the view with us. The episode was also sponsored by a company in Berkeley called Meyer Sound. They're an audio engineering and manufacturing company. They're not here to advertise or sell anything, they're just supporting the show. But there's always doing interesting things and love sharing their research going on in their labs. So if you wanna check them out, go to [meyersound.com](http://meyersound.com), there's a link down in the show notes. [music continues] And we'll be back again soon with another episode here on our Numberphile podcast.

[music continues and fades out]