

Numberphile Podcast Transcript
Episode: Making Sense of Infinity - with Asaf Karagila
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From high school drop-out to set theorist, Asaf Karagila shares his journey towards infinity.

Asaf is a UKRI Future Leaders Fellow at the University of East Anglia.

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[Ed Note: In this transcript the words this and these are freely interchanged in an attempt to capture Asaf Karagila's speaking voice, rather than follow a strict interpretation of English syntax.]

[gentle piano music]

Brady Haran [BH]: Today we're talking with set theorist Asaf Karagila, he's based in the English city of Norwich working at the University of East Anglia, there his research involves exploring the mind bending world of infinities. But before we get to these lofty concepts let's start in the desert and an unconventional route to academia.

[gentle string section music]

Asaf Karagila [AK]: So I'm originally from Israel. I grew up in the desert, so it's a small town outside Be'er Sheva, very hot very dry, but there were some forests nearby so it's this kind of very nice and pluralized kind of a background.

BH: When you grow up in the desert is it like, well, how is your childhood different from people who don't grow up in the desert. Like do you play different games and do different things as a boy?

AK: No it's just hotter and you get used to this very intense dry heat that's the main difference.

BH: What were you into as a boy? Like, when you're a young boy what were your obsessions? Were you obsessed with sport or were you already obsessed with mathematics or...?

AK: When I was like six or seven, I loved dinosaurs, you know, and then it move to like fighter jets and birds and stuff like this. But my father tells a story and you know it's not that reliable, every time I get younger in this story that, you know, when I was like four or something I already knew that if you have like one tile on the floor and you have half and you have a quarter and that you can always break it to half so this kind of understanding of infinity. I always had like a little bit math in this. My father, who's not a mathematician at all, you know, when I was six we would sit and he would show me how to do long multiplication and long division and just give me numbers and I would play with it. So mostly I was encouraged to be curious, right?

BH: What did you want to be when you grew up? Like if I'd said to the little boy, hey what'd you goin'... what job would you wanna do when you're an adult?

AK: I don't know, I think I comprehended the idea of being an adult even, you know?

BH: [laughs]

AK: I was convinced that, you know, I couldn't see life beyond like seventeen or something. Not that I'm going to die but like... that didn't exist.

BH: Yeah.

AK: But I do remember when I was I think fifteen, we had this awful maths books for school that have like fifty question that all look kind of the same, and there were these questions on induction, which is kind of... you know all of them looked like this. You have this summation of this term and this closed form, like the sum of N square kind of thing and then the closed form and you need to prove that this equals that. And I remember telling everybody in my family who would even listen, I get the idea of the proof but I want you to tell me how you got from this sum to this closed number. How do you get it? I don't know what pure maths but I think that is what pure. And they all gave me the same stupid answer, oh you have to be able to do this in order to learn what you do that. I ended up dropping out of high school, I didn't have to learn all of that but then again also the university they didn't teach me this.

BH: Hang on a second. You dropped out of high school?

AK: Yeah.

BH: If you dropped out of high school how am I... how am I sitting opposite you at a university?

AK: [laughs]

BH: There's obviously some weird... why did you drop out of high school?

AK: [sighs] So... you know, it sort of ran its course and by the end of the 11th grade I just felt like there's nothing left for me there.

BH: Were you a good student? Like were you getting good marks, or...?

AK: [sighs] Well... I was as a kid but then I was sort of a rebellious teenager and I didn't care for this as much so the things if I liked the teacher I would do well but if the teacher annoyed me I would just cut classes and never show up.

BH: Oh I like the sound of this rebellious. What rebellious things were you doing when you cut class?

AK: Nothing really. You, you know, just sit on the grass and enjoy the sun.

BH: [laughs]

AK: [laughs]

BH: Okay. [laughs] Not too rebellious then, just mild rebellion.

AK: Yeah well let's not go there.

BH: [laughs]

AK: [laughs]

BH: Okay. Okay. Alright then, so if you dropped out of high school how did you come back to education?

AK: So you can get the GED, the equivalent diploma, you can do it yourself.

You just have to register and you just go to the exam and that's it. So like a few of the exams I already did in whatever school years I could do it and the rest I just did on my own with the exception of math that I postponed for a bit later and I took like a preparation course 'cause you know, that's a bit harder to force yourself to sit down. But literature you just to read some poems, read some stories, think about them enough and, you know, you can spit out whatever they want you to spit out, so I didn't put a lot of effort and I got disengaged and that was all of it for me. I hated the idea of university when I left school. For me I was like okay I'm just gonna be like a programmer or do some computer stuff, but then I went to the army and in the army I ended up getting to some computer department somewhere. You know, they gave me the freedom to just explore the system. So I had to teach myself, you know, all these kind of servers that you don't hear about or, you know, language to communicate with printers, you know which is pretty amazing, and nobody there understood it, so I just had all the fun learning this stuff and then by the end of it I was like, okay, I'll go and study computer science and math. And then found out that there's a degree that let you do both together, but because I dropped out of school I couldn't get in, because my exam grade were like borderline and it's a relatively popular program so I couldn't go in, but people at the university said oh just, you know, join the pure math and it's the same for freshman year, so you can... if your grades are good, like, over a seventy, just you can switch and that's fine, but after one semester, you know, studying some set theory and you know this kind of abstract stuff, I was like, nope! I'm gonna be a mathematician, you know, it's gonna be.

BH: So just to make sure I understand the path properly. You joined the military, presumably that was compulsory.

AK: Yeah.

BH: It's a compulsory service in Israel. You joined the military. You got a taste for computers and computer science and thought you'd liked it.

AK: No, no, I already liked computers since, you know...

BH: Oh okay, so you already... and after doing it in the military you thought you wanted that as career, you went to... you couldn't get into that particular course at university because you had the insufficient schooling.

AK: Yeah.

BH: They said use the mathematics as a shortcut to get into computer science, but then mathematics grabbed you?

AK: Yeah, so... kind of. So, first off math already grabbed me as a kid.

BH: Right.

AK: So it just sort of reawakened this. But [sighs] the thing that the army gave me was it reminded that I enjoy learning stuff. That it's school that takes the fun out of learning and that if you have a... you know a place where you can just enjoy it then you can just run with it. And of course that if you go to university and you learn stuff and it's a lot of self-work that's fantastic, so...

BH: It's ironic though that it was being in the military that gave you that kind of freedom. People normally associate [laughs] the military with a kind of a really rigid life but you sort of...

AK: Yeah.

BH: It sounds like it was quite a positive experience for you?

AK: Well, you know it's not purely positive, of course there's a lot of downsides to it but, you know, you have to cut your hair, you know, you have to

wake up very early every day.

BH: [laughs]

AK: You know, but, yeah, and there's people telling you what to do all the time but at the end of the day I ended up in rather small place and they really appreciated the fact that, you know, I had this capacity to do things. So they just, you know, said okay, you know, just go and do those things, you know, whatever you want.

BH: Right, 'cause you were the... you were already bit of a computer nerd, so they were like let him do his thing?

AK: Yeah. Exactly.

BH: He's best left alone. So during this like first semester of mathematics when it sounds like mathematics kind of reambushed you a little bit and drew you back in, was it inspirational teachers? Was it the content, do you remember anything specific that really, just really, hooked you in?

AK: Yeah it's definitely the basics of set theory. So my professor was later my advisor during my Masters, Uri Abraham, and he's a fantastic teacher, you know, through and through, but it was just the fact that you could make sense of infinity, that you know, really sort of clicked in my head. I remember there was I think like week four or five something like this there was some homework question about something with infinity and I thought I had the answer and I talked to this girl, and she was very pretty and she said something opposite and I was like, oh okay, maybe I'm wrong. And then I found out that I wasn't. And, you know, that sort of just said, okay, you know what, I love this, I'm gonna do that.

BH: You talked when you were young about liking sort of dinosaurs and

fighter jets and those typical things that we all like when we're young. Another thing I know that does really inspire the young mind is the notion of infinity, when you're first young and you find out this thing called infinity exists. Do you remember infinity from when you were young, like, oh, what's bigger than infinity? Can I count to infinity? Or was it something you didn't really encounter until later?

AK: So [sighs] I don't really remember, to be honest. I think... like kind of the basic stuff that you know oh infinity, infinity plus one, does it make sense, does it not, you know all this basic stuff that they don't teach you about in high school. Even though it's completely possible to talk about these things with teenagers but, you know, they just don't do it. And okay I wasn't the best high school student anyway so, maybe that's why.

BH: Yeah?

AK: But... not as much. I was always intrigued by it because you know it's infinity. But you don't have the tools because until you do set theory you have no tools to really grasp what is infinity.

BH: Do you regret dropping out of high school? Was that a mistake?

AK: Best decision I've made...

BH: [laughs]

AK: ...before the age of twenty-five.

BH: Really?

AK: Hands down.

BH: Explain that, 'cause it... would you... but you wouldn't recommend it to other young people presumably?

AK: Of course I do.

[pause]

BH: Really?

AK: In fact when I was a PhD student the secretary said to me, oh you know there's this teenager who's like fifteen and he's been taking classes here and the set theory course that you teaching has no prerequisites so he wanted to take it, maybe you can talk to him and explain what's going on. And I effectively told him, look, if you're at fifteen taking university classes just drop out of high school, you have nothing to do there.

BH: [laughs]

AK: You know, and the secretary said to me, what do you do! [exaggerated voice] His mom will kill me! [laughs]

BH: [laughs] Yeah.

AK: You know, he didn't drop out, it's fine.

BH: Yeah.

AK: But... if you're smart... you have nothing to do in high school. At least in my experience, obviously other high schools might be better for you and that's great but the high school I had... and I could see my older sister, I remember this one little bit that was maybe influential in this whole thing. When she was sixteen or seventeen she was talking to a friend and she told her, look you can

find, you know, the challenge and the fun part in your studies but I, you know, I just feel being forced into this kind of stuff that I don't wanna do. And I don't like doing things I don't wanna do and as a teenager in particular you don't have this, oh, okay I'll just do this and then I can do other things. Just say no, the hell with this, I'm you know...

BH: But leaving high school can knock you off the path, can't it? To get into the cool stuff later, like university, it turned out well for you but even you encountered that problem when you tried to get into the computer science at university and they said no the door is shut to you, because of... being a high school drop out.

AK: I mean, yeah, but again, if you're smart you're gonna go to math anyway and ...

BH: [laughs]

AK: At least in Israel if you wanna study pure math and you have a pulse and an ID, you will get in.

BH: [laughs]

AK: You know, it's its own filter, you know, if you don't go through the first year, you're not gonna go with it. So...

BH: Okay.

[gentle piano music]

BH: So tell me more about this, you know, the falling in love with set theory and falling in love with mathematics at university. It sounds like it was like almost a sort of love at first sight on this reintroduction.

AK: Yeah, you know, almost yeah. I mean... you know, you just do this... basic course and it was basic to the point that is painful to me as a set theorist now to think about, oh, that was set theory. That was nothing, that was really the most basic introduction. But, you know, it was fine, it was aimed also for people in computer science and you know, they don't need a lot more than that. So it was just this but then you touch the infinity part and all of school, all through everything else, people always tell you like, oh you know, you can't deal with infinity. It's infinity. It's not a number. It's not a this. And then you have this very good professor and he tells you, well, you can in fact do that. And, you know, it just clicked in my head and it just this kind of missing piece, you know, that you go like yes, this is home... and... yeah.

BH: I talk to other mathematicians sometimes about their path through university and the different branches of mathematics they experience and how they eventually gravitated to one over another. It sounds like you were all in on set theory really early?

AK: Yeah. I mean I did enjoy other things. So I had an incredible professor who taught me Linear Algebra 2, and you know part of enjoying math, you play with the concept. They always tell you, oh you have to do the homework. You don't have to do homework. You have to think about the concepts and homework is just, you know, a medium through which you think about the stuff from class. I mean I would go to a pal and I would think like, oh, you know, so there's this vector space and it define over a field but what happens, you know, we don't really use all of it, let's try to define this over the integers. And I would give all these kind of fun names and I would go to his office hours and say I don't have questions about the course, but I came up with these things, you know, and he was an amazing professor. Unfortunately that was his first year at the university and the students just wore him down. I took a course with him every year and ever year it was more advanced course and by the third year I could see he was completely worn down in the big courses that he was teaching.

Whereas the smaller one, you know, it was always fun. So he was also very influential and he was teaching representation theory and it was an accident kind of that there was no P-adic analysis in my second year that I could, you know, join in, 'cause I would have maybe gone to number theory instead but they didn't have the course and there was set theory and I already liked that one so I just went there and you know...

BH: What is set theory?

AK: Well set theory is the way to make sense of infinity from the modern perspective. So, you know, if you go back all the way to Galileo and even before that people knew, okay, there was infinitely many natural numbers and there's the Galileo Paradox that says, well, the same amount of natural numbers so zero, one, two, three, and so on, as the even numbers, so zero, two, four, and Galileo said, well, the reason this is happening is because infinity is divine and we shouldn't deal with infinity. And that's sort of continued in this kind of fashion here and there, oh I'm sorry Galileo used the squares and the one before him Adam... of Something... I don't remember the name but...

BH: Right.

AK: It's even older, the even thing and Galileo pointed out with the squares, there's a lot of history about this. So as, you know, Enlightenment moved and rolled out and you moved from the Renaissance to this, oh maybe the Church doesn't know everything, suddenly people said maybe infinity's not divine, maybe it's something you can deal with. And that slowly progressed and you see a lot of these mathematics that is very modern coming out in the 19th century, and one of those things was Cantor saying, hang on! Let's just take all of the natural numbers as a completed object, and just work with it and we get more things and suddenly you have a theory of infinities, you know, and it's absolutely fascinating and it opened the door to so much amazing mathematics that came afterwards. It really is just about making sense of infinity.

BH: So if set theory at its core is about dealing with infinity and infinities, why is it called set theory?

AK: You want to think about sets, which is essentially how you think of a collection of mathematical objects that is also a mathematical object.

BH: Yeah?

AK: So you want to study the property of these so-called sets to see what they do and how they behave and one of these axioms says that there is a completed infinite set of the natural numbers.

BH: Right?

AK: And from that one you just get all the way up.

BH: Yeah, so that's like your... that's your ultimate set, all the numbers, that's like your... that's your dream set.

AK: Yeah. You know... that's the... that's not the ultimate set, that's sort of buying the land for your ultimate house.

BH: Right.

AK: That's the... land upon you build your castle, you know?

BH: Okay. Okay.

AK: And, you know, in a very Monty Pythonesque way the first castle they tried to build collapsed into the swamp and then the second castle collapsed into the swamp and the third one burnt down and then collapsed into the swamp but

then came Zermelo, and after him, you know Fraenkel and Skolem and other people and they build a castle and that castle held.

BH: Right.

AK: So really what you want is to understand the properties of sets, because if you can interpret every mathematical object as being a set, what you're really studying is the mathematical universe.

BH: Right.

AK: So you're not studying the sets, you're studying the mathematical universe as a whole, and you want to understand how the infinite parts of it play together.

BH: So sets can be finite and infinite?

AK: Yeah.

BH: Right. So... what advantage do sets have that I don't have if I'm just dealing with individual numbers on their own, like number theorists and things like that? What's the power of a set?

AK: So a set, you know, collects stuff to it.

BH: Right.

AK: If all you care about is, I dunno, the fact that N is a prime number, that's fine. But now you want to study all the prime numbers as a set, and ask, you know, things about that, or you if you want to move from just the natural numbers and you ask about sine and cosine and exponents and integrals and all these kind of things, you suddenly find yourself dealing a lot with sets. Oh a

function is continuous except in this small number of points. So what does it even mean a small number of points?

BH: At what point does a set give us the power to unlock all these mysteries of infinity? Or what is it about the set that gives us that power?

AK: So it's not about this set, it's about what you assume are properties of sets, and effectively what you're assuming is that there is one infinite set, namely the natural numbers, and that every set has a power set, which is the collection of all the subsets. So, you know, you have the natural numbers and the even and the primes and the non-primes and you know everything which is a prime times two to the power of six or whatever, all these kind of sets, now you can collect of these into a set, that's called a power set. This is going to give you a lot of information, and then you can take the power set of the power set and so on and so on, that's sort of where the true power [chuckles] comes to set theory, from the power set axiom.

[gentle high octave piano etude]

BH: Let's go back to university. At university did you have this same rebellious streak that you had in high school? Were you cutting courses and things like that or were you suddenly finding yourself more diligent now as a student?

AK: I mean not really, I never really did any homework. [pause] I was kind of lucky quote unquote that my first year there was a huge strike so homework were not mandatory for the whole year.

BH: [laughs]

AK: And second year and on they don't really grade homework so you don't... you just do it sporadically every now and then. 'Cause as I mentioned

before homework is not important, what's important is to engage with the material and we were just like eight, seven, eight people doing pure math. So we were always together in almost all the courses, so we, you know, we had this very fun and engaging group to discuss these things. I never really had to do that. And, you know, there's this thing that all of my teacher... all of them without fail... told us the story, oh when I was a student I would go home and reprove all the theorems we saw in class so I can really understand them. I've done none of it. I was terrified in my second year, like, I'm not doing this, what's wrong? Am I going to be good or not? And then you find out that it's not about proving the theorems from class. It's not about solving the homework, it's about engaging with it, and if you have people who also enjoy math and you sit and over a beer, your discussion is about, oh this proof or that proof and why does this work, then you end up, you know, knowing a lot more than you would otherwise.

BH: As you come towards the end of your undergraduate course, are you thinking this is it? This is my career? Or what are you thinking your career progression's going to be from here?

AK: So my father was, you know, originally a historian. He didn't get into academia very well because it's more cutthroat in his field and there was politics involved but he knew academia and when I was... not even... before I even started my first year, he told me, oh you're going to do a PhD. And I said, what are you talking about? Let me finish my first year and he said, look, your older brother studied engineering, that's point A to point B, that's how you get from A to B, there's like three different ways and you use this for that and, you know, and this for this. So, people in engineering maybe they do a Masters to specify on something, but you're going into natural sciences, they just throw you into the ocean and they tell you swim and find yourself an island.

BH: Yeah?

AK: Trust me you're going to do a PhD, you know?

BH: Yeah.

AK: So by the end of my first end I knew that I want to pursue this.

BH: Okay.

AK: Yeah. So for me there was no question, I originally hoped that my second year would be so busy that I can finish in two and a half, instead of three years, that didn't pan out but it did give me, like, the third year was very spaced out, whereas all my friends had like this extra courses I've already done so I could, you know, go home and take a nap.

BH: Yeah.

AK: So, you know, that was nice. No, I already knew... I want to do this.

BH: And doing a PhD... that locks you into academia does it? You can't do a PhD and then go and work in the stock market or something?

AK: Oh, definitely not. So when I was a... especially as an undergraduate I used to go hiking in the desert, like every two months, you know, I would just grab a backpack and go over the night to these places. And I would hitchhike, you know, back and forth all the time and one time I was talking to somebody and he said, oh what do you study? And I said, oh pure math and he said, oh you know that's great, I'm an engineer, I would hire a mediocre pure mathematician over a good engineer any day of the week because pure math is not about this one thing that you research it's about being able to make abstractions and solve problems and then applying them down, so you know, it gives you this massive toolkit of dealing with problems and that's what's important so I see a lot of my friends that gave up on academia that went to

finance, or programming, or whatever you wanna do with your life, you know, not saying that PhD in math is going to help you with everything, maybe not with cooking but it's still can do a lot for you.

BH: So... what was doing a PhD like?

AK: Fun. Kind of stress-free and then very stressful and then stress-free again.
[laughs]

BH: Yeah? So stressful in the middle?

AK: Yeah. Yeah. When I was a PhD I still lived away from Jerusalem, I did my PhD in Jerusalem and I still lived in Be'er Sheva and my advisor for Masters would go with me to Jerusalem for, you know, set theory classes that we had by advisor and his old advisor, you know, on Wednesdays usually, so he would pick me up and we would go talk either about math or whatever and one time he was also taking one of his friends to visit her mother in the hospital or whatever, and... she asks me, so did you have your crisis yet? And I was like, no everything's going great! I'm not expecting a crisis and they both laughed, they were like ha ha, you'll see. And later that year essentially everything I thought I had just completely collapsed. We found like a huge gap on the first stone that I laid and everything just [collapsing noise]...

BH: Right.

AK: ...fell down.

BH: Okay.

AK: And so... I had to spend very very stressful couple of months, you know, figuring out a way out of this. I did. [laughs]

BH: [laughs]

AK: At the end.

BH: Yeah.

AK: But... that was my... end of my third year and I remember feeling if that was my first year I would have picked a different topic, 'cause in Israel you can do a PhD for up to seven years, you know, and even five years you still get funded, so it's... you take your time just get a good result because here in the UK it's like you have three, four years [snaps fingers] you know, start from nothing go all the way there.

BH: Yeah.

AK: So you have no room for mistakes. But if you have five years you really can do it but after three years... you know, so you have to commit, you have to, you know, go through the fire, run the gauntlet and, you know, come with it at the end.

BH: I won't ask for technical details because, A, I won't understand and B...

AK: [laughs]

BH: ...this is a podcast so it makes it even harder. Who found the mistake? The stone at the bottom that had the flaw? Was it you, was it like a third party reviewer? Do you remember how the mistake was and what that felt like?

AK: Yeah, so... so I had this kind of definition for this kind of iteration of things, whatever...

BH: Hmm.

AK: And at the time there was a visitor for like the whole semester from Vienna and I knew that he kind of liked this kind of things and you know he agreed that once a week I sit for like... I see him for an hour and showing all everything and he would just point, oh okay that looks fine, just, you know, prove it for yourself in detail just so you know it works.

BH: Hmm.

AK: And there would always be like these kinds of problems and I would come back the next time and say, oh, you know, here's stuff. Same with my supervisor, you know, okay, it's fine just make sure to check the details.

BH: Hmm.

AK: And then I thought I had it fine and one of my... colleagues and friends we shared an office and shared the supervisor and we already wrote together one paper at that point [sighs] you know, I showed him what I had. And then, you know, he said, no, no, I don't agree with this claim of yours. And I said, yeah, but that comes out of the very first thing that you obviously agreed with.

BH: Hmm.

AK: And he stopped and he thinks and he says, no actually disagree with that. And then together we came up with an example that had like less than ten points kind of iteration. It was insanely simple. But so stupid. And that was just like, whoop! [pops] everything fell down.

BH: Everything went, just from what... one mistake you might have had on a bad day?

AK: Yeah.

BH: Three years earlier?

AK: It wasn't even it was a bad day it was just oversight, because I'm more of a big picture kind of idea less, you know, nitty gritty, so luckily at that point I had a lot of the big picture and I could just say, okay what do I need to recover? I need this and that and you sort of build it back up, so you have the shape of the tower but all the inside collapsed and you very quickly have to put in, you know, supports to keep it working. So I did that, it ended up overly complicated and over the years since then I've, you know, grown to say, okay, you can do this and you can do that instead and so on and so on. So it was fine at the end but... yeah, at the time it was very very intense .

[gentle violin music]

BH: Now as a, you know, professional mathematician moving up the rungs, did you take lesson from that? Has it, has it made you more thorough or cautious or careful of those foundation stones or, you know, did it teach you a lesson?

AK: I wanna say yes. [laughs]

BH: [laughs]

AK: I mean [sighs] [tsks] look at the end of the day you are who you are. And if you go and you jump and you make these, you know, very far jumps and you land then that's who you are, and if you fall you just disappear from academia because you fall too often. Not necessary that you're being pushed out, you just give up because... nobody wants to live their life just being wrong all the time.

BH: Yeah. [laughs]

AK: So you just give up after a while. And that's absolutely fine, you just say

okay, you know what I'm gonna take myself and... find an industry job that pays three times as much. What I learned this is more a lesson about myself, that I'm usually wrong. [laughs]

BH: [laughs] That's okay, is it?

AK: Yeah no, being a mathematician is mostly you're wrong and you just hope that people just remember the parts you were right. Now you don't talk about the wrong parts, right? That's work in progress, but most of the time you're at a least a little bit wrong.

BH: Obviously the stuff you publish is the stuff you get right.

AK: I hope.

BH: Hopefully [laughs] most of the time.

AK: Hope, hope. Yeah.

BH: So all those wrong moments, all those false starts and dead ends that you've experienced, you know, don't get published so the wider mathematical community doesn't know about them. Who does know about them?

AK: So, for most people they just sort of throw it in the background and, you know, I dunno, you meet someone and you sit for a beer in some conference and you ask them about this research or you tell stories like, you know, war experiences of this kind of thing, and they will tell you how they were wrong about this or that.

BH: Hmm. Right.

AK: Sometimes these things are public because you know, you go on record

and you say I think this, I think that, and then later you say okay, I was wrong because of these things and, you know, there's some record that... so I personally like to write these kind of experiences on my blog [pause] you know, which is, you know... a place for me to sort of spit it out and say, oh look it was so difficult and exhausting but how much I, you know, ended up with, so... just in my case it's...

BH: On a blog.

AK: Yeah, it's somewhere on the internet.

BH: So when mathematicians get together over a beer or a coffee or a chat, and they just tell war stories as you put it, are often those stories about things I once got wrong? You know, classic mistakes I made?

AK: Not necessarily, it's sort of, you know, it's hidden. We never say, okay, so I was trying to prove this and then we tried that and that didn't work because of this thing, so you frame this as, I learned the hard way that this approach is not gonna work because of this and that, so it's a positive thing, you actually proved something, but proved it by first being wrong about it. And then finding out why you were wrong.

BH: Yeah. Okay. So after you've done your PhD, what's the next step in your career from there? What'd you do next?

AK: So, I didn't really wanna go to the US. And I was hoping to stay around Europe.

BH: Is that a normal route, is it, for an Israeli PhD, or...?

AK: Yeah normally you're expected to do at least one or two postdocs abroad and then maybe you come back to Israel. Not everybody do but, you know, at

least if you're gonna stay in academia, you usually go abroad and then you come back. If you're really really good they will even tell you, you have a job waiting for you, just go enjoy a postdoc because, you know, it's a lot less stress, enjoy your postdoc, comeback, we'll be fine. I wasn't that good I guess, although they did tell me, oh you and the other guy I mentioned from the mistake, you two have to have good postdocs so you can come back and continue the tradition of set theory in Jerusalem because, you know, it has two of the world... the most prominent set theorists. I heard from like two or three people, well I hope that you do a good postdoc and come back.

BH: Hmm.

AK: Shortly after I left they gave a job to somebody who finished in Tel Aviv a few years earlier, and last year my friend started as a tenure track, so they're fine.

BH: Yeah?

AK: Yeah. So I don't have to come back.

BH: Okay. [laughs] The tradition continues without you.

AK: Yeah.

BH: [laughs]

AK: So, I didn't really know what I'm going to do. I applied to a few places but you know, there's this approach of, oh apply everywhere and then one of the places will take you.

BH: Hmm.

AK: But it's not a fair approach if you do something very popular... then fine,

you know, somebody will take you, but if you do set theory which is, more specialized and not everywhere has big centers in set theory, then you're unlikely if you just send applications to I dunno, the University of Oxford, they're not gonna take you, they don't have set theory. So why would you even bother them? Right? So I tried to only apply to a few specific places in the US, none of it panned out... and then I ran into the person who'd become my supervisor in Vienna, who was there again, because he came often, and I said, hey do you have some money for a postdoc? And he said, yeah I should have at least for a year maybe, a year and a half, which is, you know, it gives you some time. That's how I got my first job.

BH: Okay, so you're in Vienna now.

AK: Well before Vienna I was applying to a Newton Fellowship.

BH: Right?

AK: So, I've been to Cambridge for some conference and I've met David Aspero who works here at UEA and we started working on a paper and came to Norwich to work on this, you know, at some point afterwards, and as I was looking for a postdoc, I sent him an email, my advisor ran into him at a conference around the same week and he said, oh there's this thing called Newton Fellowship. So we applied. And everybody told me you're not gonna get it, but I got it... so that was lucky.

BH: So for people who don't understand, this is a fellowship, is a sort of a pot of money they give you to fund... fund your life [chuckles] and your career while you're doing postdoctoral studies, while you wait for that next big career step.

AK: Yeah.

BH: Yeah.

AK: Yeah. And so this is specifically a two year one.

BH: Hmm.

AK: So it's not very big but you have a decent travel money.

BH: Yeah.

AK: So, it's a nice postdoc.

BH: Yeah.

AK: I applied to this, I was already... I talked to the guy from Vienna, to Martin and he said, it's absolutely fine if you leave early...

BH: Right.

AK: ...no hard feelings because you have to be upfront about these things.

BH: Yeah, yeah.

AK: Yeah.

BH: So the Newton Fellowship that you had been given to spend two years as a postdoc, what university was that going to be at?

AK: So that was here at UEA.

BH: Okay.

AK: Yeah.

BH: The University of East Anglia, that is.

AK: Yeah.

BH: Yeah.

AK: In Norwich. Which is, you know, a pretty decent size set theory compared to other places in the UK.

BH: Mhm?

AK: I dunno. So this was a very, active set theory research area.

BH: Yeah.

AK: So it was good place to go for a postdoc.

BH: So for two years you're doing a postdoc here at UEA with this Newton...

AK: Yeah.

BH: ...fellowship. What do you do, because you're kind of... you're not the boss. You've got a... you've kind of got a boss, but you're also not like the bottom of the heap anymore. You're not like a PhD. When you're doing a postdoc in mathematics, what are you doing? What work are you doing and who for and to what end?

AK: There are two types of postdocships. There's, you know, being a postdoc for someone.

BH: Hmm.

AK: That you are employed on their grant and you do their research.

BH: So they've got a big project and they might have helping with grunt work or...

AK: Exactly, yeah.

BH: ...side projects. Okay, yeah.

AK: Now they could also say, I don't need help on this project, just do good math, whatever you want, but it's still on the project.

BH: Okay.

AK: And there's fellowships that essentially say, okay, you write the project, here's a bunch of money, go work on this project for the next how ever many years.

BH: Okay so you're... more your own boss?

AK: Yeah.

BH: Right.

AK: Yeah, so you have a host, it's somebody who's supposed to sort of oversee you and mentor you and all this stuff but you know, you're kind of your own boss.

BH: Okay.

AK: We had this project about something that's slightly different from my PhD work, because everybody I met kept telling me, oh you have to do something different because if you just work on your PhD stuff then, you know, you don't show... any kind of breadth of... you know, subject and knowledge and whatever. So I wrote something else and... we got the... but... my real passion, whatever I did in my PhD, so every time I would sit alone in the office, I would go and work on that. You know, we worked a bit on our stuff and I would work a lot on that stuff, and that was fun, but, you know, it's only two years, it's a bit stressful, oh, at the end they ask, oh what would you do different? And I said, three years. That's...

BH: [laughs]

AK: ...you know, because...

BH: Yeah.

AK: You have to start finding a job almost as soon as you start.

BH: Yeah.

AK: That's really... that's crazy. Yeah and then afterwards I just started applying to universities, I applied to Cambridge and you know obviously didn't get it, a few other places. Then the university said, oh there's this UKRI Future Leaders Fellowship thing, you now, you can try to apply for this. So... I started working and I got a lot of help here at the department and at the university level they really helped me to make this into a really nice proposal so you pass the internal sieve gate and then, you know, you submit the application and you wait and you wait...

BH: Yeah.

AK: ...and you wait some more [sighs] and then you wait some more and then... I was giving a talk in Manchester and the morning after I see my email, you know, you got back the reports, you know, the reviewer comments on the proposal. And you send it back and then you wait and you wait and you wait and they said that the announcement of who's invited for interview are going to be on the first week of December, and it comes and it goes... and your nerves are plucked completely.

BH: Are you like checking your email every twenty minutes or what are you...?

AK: About five minutes. You know?

BH: Right [laughs]

AK: It's like [tapping]

BH: Reset.

AK: Refresh, refresh, all the time, refresh!

BH: Yeah.

AK: Nothing.

BH: Yeah.

AK: And you go like, okay, should I send them an email? Should I send them an email? And my head of school said look if they said the week you have to give them the whole week.

BH: Yeah.

AK: Monday morning you can send them an email.

BH: Yeah.

AK: The next Monday arrives and I said... there's like about five hundred people probably applied, I don't need to send them an email, they know they're late.

BH: Yeah.

AK: And a few hours later, you know, I get the email and it says I've been invited to the interview and I, you know, immediately run and shout and...

BH: Yeah.

AK: ...you know, very happy. I start reaching out to people I know that have this to ask for tips, you know, do practice interviews, at least two, and get a diverse panel for this kind of practice and so on and so on and you work and you, you know, workshop your seven minute presentation and they give you a list of optional questions and what people kept saying, and that was absolutely true, is that if you got to the interview they want you to have the money.

BH: Right?

AK: You only competing against yourself at that point.

BH: Just so people understand the stakes here...

AK: Yeah?

BH: ...what are the stakes? What's on offer here, what are you asking UKRI to

give you?

AK: So the offer, and they slightly modified it in subsequent rounds...

BH: Hmm.

AK: ...was a four year fellowship...

BH: Right.

AK: ...that you can ask for three extra years.

BH: Right.

AK: And you also get a permanent job, with it.

BH: Right.

AK: And apparently they change it so that it's not guaranteed to have a permanent job because some of the big universities don't like to, you know, hand out jobs like that, so they changed the wordings, but for my round it was still part of the deal.

BH: At worst you're looking at maybe seven years of...

AK: Yeah!

BH: ...well funded research.

AK: Extremely well funded research, yeah.

BH: Yeah.

AK: And I asked for money to have two postdocs and conferences and a lot of travel.

BH: Mhm.

AK: So you know, very well funded research center essentially for four to seven years.

BH: Yeah, yeah.

AK: Yeah. And on the other hand, I had no other job offers.

BH: Right. [laughs]

AK: So it's this or nothing.

BH: But how did the interview... what was it like doing that? It must be like going before a firing squad?

AK: [sighs] Almost like the first two minutes you really feel like that, you know?

BH: Mhm.

AK: You have to... you don't have to wear a suit but you know you kind of... feel that you need to wear a suit. You know, I was talking here with...

BH: Did you have your hair back in a ponytail or...?

AK: Yeah, yeah.

BH: Right, right. Very respectable. [laughs]

AK: I can't wear a suit with, you know, the hair just flowing to the side and...

BH: Rock n' roll hair, yeah.

AK: So I talked with one of my colleagues here about this, wearing a suit, and he said well you don't have to wear a suit but if you don't wear a suit you're going to have to convince me you're a really good mathematician because you're not taking this seriously enough.

BH: Right.

AK: I don't know if I agree with this, I don't think everybody I spoke would agree with this approach, but what do you have to lose? Just put on a suit.

BH: But the thing is, from everything you tell me, from even when you were in high school, it sounds like you're someone who doesn't like to be told what to do and you oughta do it your way. I can imagine you walking in jeans and a t-shirt and saying, take me or leave me, this is what I am. But you conformed on this one?

AK: Yeah I mean, [sighs] the first time I ever wore a suit was just a few months before that. You know?

BH: Right? [chuckles]

AK: For a wedding that I went to. So... [laughs]

BH: [laughs]

AK: You know, it was a new experience and it wasn't that bad.

BH: Yeah?

AK: So I figured, okay, well, why not?

BH: Alright.

AK: I even had a Formal Friday, 'cause I would normally just walk barefoot or just, you know...

BH: Mhm.

AK: Basic shirt and jeans whatever, and I would just come with a suit every Friday. [laughs]

BH: Alright. [laughs]

AK: Yeah to the university just to be more comfortable wearing a suit.

BH: So it was like practicing wearing a suit?

AK: Yeah, yeah, exactly.

BH: [laughs]

AK: It was fun, I liked it.

BH: Yeah? Yeah.

AK: I kind of miss wearing a suit.

BH: Yeah?

AK: I almost wore a suit today.

BH: Oh wow, we've been doing some Numberphile videoing today so that would have been quite interesting. [laughs]

AK: Yeah, well, I didn't but, you know...

BH: Yeah.

AK: It crossed my mind.

BH: Alright.

AK: Yeah so, you know, you go to the interview, I did this terrible mistake. So they tell you, you can come to a hotel, we can even reimburse you for this hotel, and I chose the same hotel as the interviews.

BH: Oh.

AK: Now that sounds smart because you say, well I just come down and go to the interview.

BH: Yeah.

AK: But it's super stressful because it's a lot of interview. So all of the hotel was the interviewers of the different panels.

BH: Ah.

AK: When I checked in I went up in the elevator and this woman asked me, oh are you here for the UKRI thing? And I was like, yeah. Oh which panel are

you sitting on? [laughs]

BH: [laughs]

AK: I was, what? No, I'm... I'm interviewing. [laughs]

BH: Alright, okay, yeah.

AK: So it was very stressful like after interview I went for dinner with my colleague in Bristol and on the elevator I meet one of my interviewers.

BH: Hmm.

AK: And we talk a bit... and he said something that give me the impression like I didn't do very well.

BH: Right. [chuckles]

AK: And I was like, oh my god... he would only say that to me if I'm not getting it. Oh no! You know, it was was extremely stressful.

BH: Yeah.

AK: I would not recommend this.

BH: Okay.

AK: You know?

BH: How long after the interview did you have to wait for the news?

AK: So the interview was... the very end of January and they originally said,

two weeks. And then they said, three weeks. And then they said, first week of March. And they actually kept it. So it was Monday morning and I had to send something back or pick something from some, you know, Hermes shop or whatever and I go out of the house at nine and I come back up just to pick it up and I said, oh I'll just check my email, it's probably not there, it's probably gonna be another week. And... exactly three seconds before they send me a congratulation email.

BH: Oh Nice.

AK: Do you accept?

BH: Yeah.

AK: And I immediately just, yes, yes, yes, yes, yes, yes, yes, of course I accept!

BH: [laughs]

AK: [laughs]

BH: Yeah, yeah, yeah.

AK: So...

BH: So now... you're set now for quite a number of years of...

AK: Yeah.

BH: ...of security and funding and the ability to...

AK: Yeah.

BH: ...push your research forward.

AK: Yeah. And this fellowship comes with a lot of nice perks like I don't have to teach.

BH: Right?

AK: Yeah, so, I can really focus on the research...

BH: Yeah.

AK: ...at this point. And I mean... I was going to live on an airplane for this two years, right? Just fly to here, work a bit, fly to there, invite people over, unfortunately we can't do that.

BH: Yeah.

AK: Yeah. But I can use the money for a big conference in three years when everything is hopefully back to normal.

[gentle string section music]

BH: So you mention that you don't have to teach, obviously a lot of people at universities have a component of teaching students and a component of doing their own research. You're in a situation where your grant means you don't have to teach, but from what I know of the little bit I know of you [chuckles] today even, you really are enthusiastic about outreach and communicating your subject and things like that. Are you saddened to not be teaching? Do you... would you like to be teaching students?

AK: I was teaching all through my Masters and my PhD, I even... so normally you do like TA work. And initially...

BH: The TA's teaching assistant?

AK: Teaching assistant, yeah. And initially that means that you actually teach.

BH: Hmm.

AK: So you do revisions of what was in the class and you solve things but you also prove, you talk about the theory a little bit. So it's a lot more hands on than what maybe happens here where they just sit in a room and people come and ask for help.

BH: Mhm.

AK: You know, doing the homework. And so I've done a lot of teaching. And so... I felt it's fine if I don't teach for the next couple of years.

BH: Okay.

AK: Now add to this the fact that because of the Army service in Israel most of the students are older. They've seen the world because... nobody goes to university after the Army service. They take a year, two years, they work, they travel, they sort of unwind from this very stressful experience, so they come to the university, they're already very much adults, it's not high school plus.

BH: Hmm.

AK: So the level of students is incomparable to anywhere else in the world.

BH: Right.

AK: Again, just about the maturity and stuff, I'm...

BH: Hmm.

AK: ...not saying that they're necessarily better than other students.

BH: Yeah.

AK: Just, you know, the starting point is a lot better.

BH: They're more worldly wise and...

AK: Exactly, yeah.

BH: Yeah.

AK: So that was all the fun, and then you come here and you see how people complain about students thinking that this is just extension of high school, you know, all of this feel that it's a bit of a... you know... a mill to produce degrees. It's not, but you know, sometimes it feels like it.

BH: Hmm.

AK: Sometimes it's feel like oh [sighs] why am I even teaching this. I'll give you this terrible example that I'm trying to... make people upset about. You have to write the exam before you start teaching the course.

BH: You as the...

AK: As the lecturer.

BH: As the lecturer, yeah.

AK: Yeah.

BH: Yeah.

AK: Now they say that this is, you know, to make sure that everything is fair for everybody.

BH: Right.

AK: But how is that fair. I have seven years of teaching experience and no two years were alike because I would teach the same course like four years and then a different course for four years, so no two years are a like. Every time students have something that they saw on another course that, you know, sort of primes them for a specific subtopic, you know, to be more interesting here or you put more focus there. If I have to write an exam before I even engage with the students, how is it fair for them? I'm denying them this kind of curiosity.

BH: Because you'll have to teach to your exam, rather than...

AK: Yes! Exactly.

BH: ...the natural drift of... yeah.

AK: I covered my host a couple of times when he wasn't here and was like, oh maybe you can cover one of my lecturers in the set theory course. And... one time he said, you have to teach this proof, it's going to be in the exam. That is insane! That is crazy. The reason they do it, I was told, is that some ten years ago or so there was a strike just before the exam period and the university ended up without exams.

BH: Right?

AK: So now they want everybody to write the exam and then if you wanna strike and not grade and... you do you, you know?

BH: Okay , right. [laughs]

AK: We just get somebody to read your solutions and grade the exams.

BH: Okay.

AK: So, that's really godawful.

[gentle string section music]

BH: What do you want to achieve in the next maybe seven years? Is there some huge problem in set theory? Is there a million dollar proof that you wanna find or is there like... what's your holy grail now? What are you working on?

AK: So one of the main focal points of set theory is adding axioms to your theory. So you have sort of the base theory for mathematics and then you say okay what happens if we add these axioms that increase the... how strong is the theory? You can prove more now.

BH: So axioms are kind of like these underlying statements that are assumed to be true.

AK: Yeah.

BH: That you build all your mathematics on?

AK: Exactly.

BH: Right.

AK: So, and you're free to choose different ones and do different work but you want to ask how much can you prove with certain axioms and this lends itself to this very nice hierarchy called Large Cardinal Axioms. So if the basic one set of all natural numbers is a set, that's, you know, it's called the Axiom of Infinity, and Large Cardinal Axioms are often called higher infinity. 'Cause they sort of take these properties that you have on the natural numbers and you say, okay now let's add this kind of thing but higher up. And one of the main axioms we use all the time is the Axiom of Choice.

BH: It's a very very famous Axiom, that you and I have spent the last two hours discussing.

AK: [laughs]

BH: [laughs] And I'm no closer to understanding it. But I know it's famous and important.

AK: Yeah.

BH: Yeah.

AK: So, a lot of people want to know, what happens when you take it out?

BH: Right. Is it necessary even?

AK: Exactly.

BH: Right.

AK: So for somethings we know it's necessary but you can also ask how necessary to what kind of fragment of the Axiom of Choice do you need to keep

in order to prove a certain thing.

BH: Okay.

AK: And the Axiom of Choice let's you look at this Large Cardinal Axioms and say oh I have like three or four different ways to phrase this axiom and they're all equivalent but this equivalence is very much dependent on the axiom of choice, now some of these were studied extensively in the Eighties, so you say, okay I'm not assuming the Axiom of Choice just this one way of phrasing but it's sort of the wrong way to think about this Large Cardinal Axioms and until a few years ago there were no serious works on, let's look at the actual [sighs] large part of that, you know, of the Cardinal. So I had some work about this with my friend from the PhD, we had this, obviously I think, that a fantastic paper where we break ground...

BH: Hmm?

AK: ...and essentially create this whole new field and what I want to do is sort of push that forward and see how far I can get with it.

[Gentle chimes]

AK: So there's a problem that's open since 1902. When I was a Masters student I read about this. And I said to myself what do you need to prove, you know, to solve this problem one way or another? That's essentially the thing that guides me to come up with new ideas through and through and...

BH: What's that called?

AK: It's called the Partition Principle.

BH: Right?

AK: And the problem is whether or not it implies the Axiom of Choice.

BH: Okay.

AK: So, without getting technical you can phrase the Axiom of Choice in a certain way and then the Partition Principle says, okay just, you know, remove this one tiny bit from the formulation, is it equivalent? We don't know.

BH: Okay.

AK: So there's been some recent progress about this. A group of people from Brazil, none of which is a set theorist.

BH: Hmm.

AK: Developed this new thing called Flow and they claim that, you know, they proved that it doesn't imply the Axiom of Choice. The first paper is available on archive, it had a lot of gaps. The second paper was released two months ago, I am still reading through it. I just had... some crazy two months so I had to put it aside. But I hope to get back to it, last week...

BH: Right? So this Partition Principle is it... that's your kind of like, your white whale, the Moby Dick, is it? It's like your thing you'd love to... crack?

AK: Yeah. Yeah. Kind of.

BH: Yeah?

AK: And even if they do prove this, you know, they use this kind of very non-standard methods and its still interesting to say, okay, let's develop the standard set theory methods to the point where you can get this proof.

BH: Is it a really famous problem? Like would everyone in set theory know about it or is more just your problem? Or is it like... is it the big trophy that everyone in set theory talks about?

AK: So... people don't really know about that. In fact, a lot of people who teach set theory without knowing a lot about the Axiom of Choice would just assume that this is just the Axiom of Choice.

BH: Right.

AK: And that's equivalent and that's it and there always surprised, oh! You told me about this thing, oh really it's not equivalent? Oh it's open. But it is the oldest currently open problem, you know, in set theory. So, you know, I think it's at the very least interesting, right? And if you look at Fermat's Last Theorem, it's not a very important theorem, but on the journey to, you know, to get there so much amazing new mathematics was developed. So even if this is not a very interesting problem, you know, on paper, I think that to get there you will need to develop this very good tools and then you can apply them elsewhere.

BH: Alright, well I'll come back in five to seven years...

AK: [laughs]

BH: ...and you can tell me how you did it.

AK: Well we can do an interim, you know, meeting...

BH: Alright [laughs].

AK: ...in two years.

BH: Alright, two years.

AK: Yeah. [laughs]

BH: [laughs] Good. You described to me earlier that set theory is right at the frontier, like at the extremities of where you can go, which you put it in a really nice poetic way. Are you just being really biased or would most people say set theory's right at the limits of...?

AK: I think it's a bit of both. I mean, a lot of people will tell you, oh it's just philosophy. It's not even real math anymore because they don't do real things, you know, they deal with infinities and this crazy nonsense. But, that's not true. It's pure math. [pause] It's just... at the end of how abstract you can be and still do pure math. You know, just in favor of completeness there are other foundations of mathematics you can go to and they're also sort of at the end... at the frontier and there would be people who tell you, oh don't study this because all math is going to be based on, I dunno, homotopic type theory [piano music fades in] in five years. You know? But that's not true.

[music fades up and continues]

BH: Well that's all for today but please see the notes for today's episode for more links and information. [music continues] Thanks to the Mathematical Sciences Research Institute in Berkeley, California for its support of Numberphile. And also thanks to the UK Research and Innovation Future Leaders Fellowship for their help with this episode. [music continues] I'm Brady Haran, and you've been listening to the Numberphile podcast.

[music slowly fades out]

