Numberphile Podcast Transcript
Episode: The Badly Behaved Prime - with James Maynard
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James Maynard is making waves in the world of prime numbers - we spoke to him in his office at Oxford University.

James is appointed a professor

And wins the Cole Prize

James discusses the Twin Prime Conjecture - Numberphile video

Small Gaps Between Primes - paper

Large Gaps Between Primes - paper

Large Gaps Between Primes - Numberphile video

James on his love of primes - Numberphile video

James Maynard playlist

Prime Number playlist

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[gentle piano music]

Brady Haran [BH]: James Maynard is one of the young guns of modern mathematics. [music continues] Specializing in prime numbers, he made his name for pioneering work on one of maths most famous problems, the Twin

Prime Conjecture. [music continues] He's since made breakthroughs on a number of other problems and he's accumulating an impressive collection of prizes along the way, in fact, since we recorded this episode, the American Mathematical Society announced he's won another. They're giving him the 2020 Cole Prize in number theory. [music continues] Not bad for a thirty-two year old who, by the way, has already been a full research professor for two years. I met James in his office at Oxford University.

[music fades up and out]

BH: Let's start at the beginning, little boy James. Were you like… were you gonna be a mathematician? Were you always like mathematical?

James Maynard [JM]: Uhh…

BH: Were you a prodigy?

JM: [pauses] Okay I definitely don't think I would have counted as a prodigy. It's very interesting looking back because at the time I never… I was always quite good at maths. Maths was probably my favorite subject at school most of the time, in school. So, it's easy to look back and say, oh clearly I was gonna be a mathematician the whole time. Somehow it was written in stone, but it certainly never felt like that when I was a kid. It was definitely never clear to me as a young age that I wanted to be a mathematician or nothing like that or even that being a mathematician was a job that, yeah, sort of when I was choosing the school subject and things at each stage, okay yeah, maths is one of my favorite subject so I guess I'll continue doing maths, but it wasn't that, yes I definitely want to be a mathematician or I'm clearly going to be a mathematician or, you know, maths is the one and only thing for me.

BH: Was it a favorite subject of yours just because you were good at it and found it easy or because you got pleasure from it. You liked it?

JM: It's very difficult to say when you're a kid sort of you definitely like things that you're good at, I think. Yeah, I think it was a bit of both, that I definitely liked it a bit more because I found it more straight-forward and I thought I was good at it and I'll get high marks in things in maths without too much effort so that's one thing that I liked about it. But I definitely also genuinely liked just thinking about the ideas. So, I remember there was one time when it was maybe I was in primary school I think, that I had music lessons, I was learning the violin, I was pretty pathetic at learning the violin but I was learning… I was struggling either way and for one year I was particular pathetic at learning the violin because my violin lessons were always directly after my maths class and I was constantly just thinkin' about the maths that I'd been doing in the maths class when I was supposed to be playing the violin with my violin tutor and yeah, this meant that I was just completely pathetic that year learning anything on the violin.

BH: So you did have this curiosity though to know more than what just was in the textbook and what you did in the lesson? You were like, oh I like these ideas I wanna go a bit deeper.

JM: Yeah, and certainly… the more went it on the more I got more and more into trying to think about these things more independently and go a little bit beyond the standard set of books and things, so yeah when I was in primary school it was more just thinkin' about what we'd done and thinkin' about the ideas and trying to understand them a bit better and things like that. As I grew a bit older it was being a bit more adventurous and think how, hey… what's the extent of these things, what are other questions you can ask?

BH: Did you have those other interests? Did you wanna play cricket for England or be an astronaut and things like that or…?

JM: Oh, yeah, you bet! Um… there were lots of different… aims for my life and mathematician was never one of them when I was kid so…

BH: Well gives us a few. What were a couple of them?

JM: Umm.

BH: If I went and met the little boy and said what'd'you wanna be when you grow up? What would the likely answers have been?

JM: Uh… so… there was one phase when I definitely wanted to drive a tractor.

BH: [laughs]

JM: I wanted to be an astronaut for a bit. I remember I had a big dinosaur phase and I wanted to be a paleontologist. So I think that would have maybe… already singled me out as a slightly odd kid if you go and ask them what they wanna be and I say, oh I want to be a paleontologist.

BH: I dunno, I reckon there's a lot of dinosaur obsession these days among kids…

JM: Yeah.

BH: Yeah.

JM: So, yeah, I think particularly maybe it was when I was a younger kid I would be like dinosaurs and then I had another dinosaur phase around when Jurassic Park came out, I think.

BH: Of course. Of course. Obviously you are like Mr. Prime Numbers these days, do you remember your early encounters with prime numbers? I mean they're something you encounter quite early at school, aren't they? Did you have a fascination with prime numbers when you were young like you do now? Or was it just part of the whole math conglomerate back then?

JM: Again it's very difficult for me looking back to be honest. That my sort of initial answer would be that I didn't think of primes as particularly different or particularly special but having said that I remember for my university applications I explicitly put down that I was interested in number theory for my university application, despite the fact we weren't taught number theory at school and I'd just sort of learnt it independently and so there was clearly something going on that was sort of fostering a fascination with number theory and prime numbers from an early age even though again I think I wasn't so aware of it at the time, that I was, maybe fascinated by these things without really be aware of it 'cause for me it was just all sort of fun different logical ideas.

BH: Were you surrounded by mathematical people? Was in your family mathematical or like was it something you just picked up yourself or…?

JM: Yeah not at all really. So both my parents were linguists, they were both essentially French and German teachers, so we'd go on holiday quite a lot abroad but there was no one in my family who was explicitly mathematical or even scientific. Yeah if you'd look at my family I'm definitely the black sheep of the family, the rest of them are all linguists and historians and things and I'm the one scientist so I stick out like a sore thumb in that sense.

BH: At family gatherings like is it hard for you to talk about your work and interests, like are they extra interested 'cause you do the different thing or are they a bit less interested because it's sort of not their wheelhouse?

JM: There's a sort of niche interest where there's a very brief joke about me testing them to see if they can tell someone else the sorts of things that I work on… but normally once I've made them sweat a little bit after that then it's accepted amongst everyone that we're not really here, a family gathering, to talk about maths, there's other things that we talk about then.

BH: So let's role-play for just a second, if you… if you were like your mum or your dad and I said, what does James do for a living? Tell me what you think their answer would be.

JM: Well… uh… uh… he works on prime numbers so it's something to do with the pairs of the primes that are close… together… once or… maybe… infinitely close together…

BH: [laughs]

JM: Something like that… yeah. Prime numbers, close together.

BH: They'd know what a prime number was though?

JM: Uh… okay I'd hope so.

BH: [laughs]

JM: [laughs] Maybe that'll be the next question, the family gathering, can you define a prime number for me?

BH: [laughs] Alright then. And just kind of in a condensed way before we get to you… to kind of where you're at, at the moment, obvi… what's… I'm assuming you got good marks in high school for mathematics? Do you perform well?

JM: Uh… yeah I think I was good. I was not one of these child prodigies who was miles beyond everyone else but I was definitely getting good marks in maths, I would say.

BH: And then where did you sort of study and, you know, go to university after school?

JM: So I studied at Cambridge University.

BH: Hmm?

JM: At Queen's College there, and… yeah that was when I sort of got more into sort of proper maths in a certain sense. That there's a fairly big difference between the sort of maths you learn in high school and the sort of maths you learn at university and so I think the… rigor of the logical ideas and things was when I sort of really was quite keen on, yeah, maths is the sort of subject for me.

BH: It really increased your passion, it wasn't like, oh this is different to what I expected? But it was, but in a good way?

JM: Yeah. It was different from what I expected but in a good way, exactly.

BH: And then when you finished at Cambridge you did like a PhD?

JM: Yeah that's right. So I then came over to Oxford to do a PhD, did a PhD here in Oxford and then after that I had a few postdoc positions so I spent a year in Montreal. I spent basically a year in the US, partly in Berkeley, partly in Princeton. But otherwise I was also back in Oxford for quite a lot of the time.

[gentle violin music]

BH: Anyone who kind of like follows what's been happening with Twin Prime Conjecture and gaps between primes and this research that's been going on, it's kind of portrayed, you know, in the news or on the internet that since Zhang made his original breakthrough, it's been portrayed as this team of really amazing mathematicians, Terry Tao and a few others, you know, really doing great stuff, and you're sort of portrayed as this one guy on his own, on the other side like sort of working in parallel and you're quite friendly with them but it's always sort of portrayed as this… Manchester United superstars and this one guy on his own who's also like having all these amazing breakthroughs. Is that like… is that the reality of it? Does it feel like that?

JM: Uh… yeah not really at all. So, I guess they were working on optimizing lots of the really nice ideas that Zhang had come up with and I was thinking about somethin' that was a slightly different approach that I'd been thinkin' about from before Zhang's work and things and it turned out my thing worked out in a way that it was way better than I had possibly expected and was therefore very relevant for the stuffs that they'd been thinking about. But then I joined their team and so… it was more as if I was recruited into Manchester United than… I was… the League One side that was beating Manchester United. [laughs]

BH: Alright it wasn't David and Goliath, then?

JM: Yeah.

BH: It was more like…

JM: No, yeah, it was me teaming up with Goliath to author anything else.

BH: Okay, so these days, it's all kind of one big group now is it? It's not like they're doing what they're doing and you're still working on your alternative way?

JM: Yeah so maybe the… big Polymath project that we had as this collaborative project to try reduce the gap down to small as possible has ended now. So I don't think there's a team that's actively workin' on the problem because we seem to have reached the natural limit of most of the ideas and to really make big progress we'd need to have a big new idea, but I think this is also very representative of how mathematics works. That, in some ways it's… very collaborative you have lots of colleague support things and you're trying to… come together to… mesh ideas together and then… understand new ideas when they come out. But equally there's lots of individual discovers that happen and that changes slightly how you think about things and you adjust to the new ideas to try and incorporate them and try to bring as much together as possible to always get the best results.

BH: So just so I'm completely across where the Twin Prime Conjecture is now, this is this… this is this elusive proof that there are infinite number of primes separated by just two.

JM: Yeah.

BH: Where's that now? Where's it your jammed at a higher number, aren't you? There's an infinite number of primes but they're separated by…

JM: Yeah, no more than two hundred and forty-six. So we know there's infinitely many pairs of primes that differ by at most two hundred and forty-six. So if we could bring two hundred and forty-six down to two, we'd have proved the Twin Prime Conjecture. But unfortunately we're pretty stuck now at two hundred and forty-six.

BH: Is this something that… people and in particular you are still working on? Is this like your… number one… thing you think about when you go to sleep at night or have you… you're moving on to other things like… where is this at now? Is it gonna get to two in my lifetime?

JM: Uh… okay so, so… fool's game to ever guess how long a conjecture…

BH: [laughs]

JM: …should be open…

BH: [laughs]

JM: …for and when things are gonna get proven. I remember… one person telling me that if someone ever asks you how long will it be until we prove a certain conjecture then you should always say, well the length of time that a conjecture's been open for because if your just randomly distributed in time you're most likely to be around the middle of when a conjecture is. But that doesn't even work for the Twin Prime Conjecture 'cause there's… people don't really know how long it's been open for. It's certainly over a hundred years old, but some people have speculated it could be thousands of years. So even if I was tryin' to use this way of suggesting how long it should be open for… it's not clear what the answer should be.

BH: It could be thousands of years!

JM: It could be thousands of years for all we know.

BH: [laughs]

JM: That again this is another feature of mathematics that somehow we make big breakthroughs but then we get stuck and we're stuck in potholes for quite a longtime. It's clear that for the Twin Prime Conjecture you definitely need some big new idea from somewhere, so in my day to day research, I'm thinking much less directly about twin primes or gaps, small gaps between primes now, and thinkin' more about developing other techniques for understand the primes in different ways that maybe ultimately would lead to some new insight that could be combined with the previous ideas to get results about gaps between primes.

BH: Is Twin Prime then like a touchstone for you? When you're… every time a new technique is developed do you go back and think, does this help with Twin Primes? Or is it not like that? It's like you don't… like… I'm trying to think is this like a white whale for you? Is this like what Fermat's Last Theorem was to Andrew Wiles or is not… a big deal to you?

JM: Okay so… yeah the Twin Prime Conjecture has a sort of special place in my heart and so… yeah it would be amazing for me if someone proved it in our lifetime that, it's sort of my favorite problem in mathematics I guess. However… I guess… unlike Andrew Wiles there doesn't seem to be any clear route to proving the Twin Prime Conjecture, it really needs some radically new perspective from somewhere and so because of this I tend not to directly think about the Twin Prime Conjecture so much but more a circle of different easier problems where hopefully we can build up enough of a toolkit to make progress. But whenever someone does come up with a new idea suddenly I guess we have a new hammer basically and everything looks like a nail, so it's very natural for me to think through a whole list of different problems that I have in my head and to say, hey, we've got this new idea can this idea help with any of these different problems and so the Twin Prime Conjecture's always on that list even if maybe it's a bit ambitious to ever hope that one smallish new develop could really make big progress on Twin Primes, it's certainly one of the things that I mentally test against.

BH: Can I ask you about the age of mathematicians? Because you seem quite young to be an Oxford professor. I don't know if that's true or not but I mean obviously the success you've had has resulted in you having like quite a nice office here and a good job so congratulations to you.

JM: Thanks.

BH: But you seem quite young, but then we also hear that… it's… I dunno if it's a cliche or true, that the best mathematics is done by people when they're young. Perhaps perpetuated by the fact they give the Fields Medal to people who are under forty. What do you think about that belief, that you're probably at your absolute peak now?

JM: Okay I… I'd like to think for myself that's it not just downhill from here…

BH: Yeah. [chuckles]

JM: …and I don't really buy this idea that mathematicians do their best work when they're young that there's always a slight advantage of young people in the sense that their minds aren't set into ruts in the same way that you naturally get when you've thought about a problem for quite a long time and so sometimes new people can come up with very different perspectives on things. So often when you have a new mathematical breakthrough it's very much someone from who's not super familiar with the field who's comin' in from slightly outside and that could be because they work in a different field or could it be because they're relatively junior in the field and so… they haven't got… the same perceived wisdom and so they think about things in a slightly different way. So, there's that one advantage but I definitely wouldn't buy this idea that mathematicians are at their peak when they're… less than forty and they are on a gradual decline after that, I think there's lots of really top level mathematicians who work well into their sixties and beyond and much of their best work happens later on in life.

BH: That makes me feel a bit better 'cause I always feel really guilty when I ask you to do an interview 'cause I feel like, maybe I'm eating into a few hours of the…

JM: [laughs]

BH: …of the few hours you've got left of… [laughs] solve the Twin Prime Conjecture. [laughs]

JM: [laughs]

BH: I feel a bit less guilty.

[gentle violin music]

BH: I want to get some idea how you do research. I always hear like… ‘cause as professor you have teaching time. I understand what that means. You’re in a lecture theater teaching or you’re with a group of students talking to them. But then I know you also have research time. Which is the most valuable time to you, I know. But what does that look like? How do you do research? If you said to your friends or someone, okay I’m spending a day doing research day, it’s a research day, what does that look like? Does that involve having a bath and thinking in bath full of hot water? Going for a walk around Oxford? Sitting at your desk with a piece of paper and a pencil like… what does research look like when you’re trying to have a breakthrough?

JM: Yeah so most of the time it’ll be me working at my desk with a pen and paper and there’ll be some mathematical idea that I’m working on and I have a project on at the moment that I basically don’t understand at all. And I’m just trying out different ideas to test what on earth is going on. That somehow… a huge…

BH: So like… so like calculate… are you sitting there calculating or manipulating equations or…?

JM: Yeah so I’ll be playing around with equations and things on pen and paper and sort of writing down rough ideas and maybe trying to visualize things so it’s often a very good tool if you can somehow draw a picture. Somehow that seems to me at least to help me try and understand what’s going on, but I’ve certainly heard mathematics described as you sort of spend six months blind in a dark room fumbling around trying to look for a light switch and you just spending the entire time trippin' over chairs and things and then eventually you find the light switch and everything is straightforward and you go into the next room and then it's another six months bumbling around in a dark room. And so I'm very much doing this chaotically trying to find… understand what on earth's gonna on with these things I don't understand by manipulating equations and looking at toy examples of the problem and most of them is pen and paper in my office.

BH: Is there anything you do to create a better environment? Like do you take the phone off the hook, do you switch off your computer? Do you play music? Do you have to have a glass of Coca-Cola? Like what are the things that you do that will… that help you?

JM: Umm… I guess I have a few different sort of little tics if you like that… I find that going for a walk often helps clear my mind a bit that… it gets pretty depressing if you've spent three hours straight sort of intellectually bangin' your head against a brick wall and so going for… just a wander around the maths department…

BH: Just the building though?

JM: Yeah just the building, it doesn't have to be a long walk but I find it helps sort of clear my head a bit. I drink water semi-obsessively as a way of sort of trying to clear my head again in some way. So lots of it is things like that… that I… yeah, walking and trying to distract myself a bit because there's some very strange process that I never quite understand where I think there subconscious is doing a lot of the thinking for you. So I'm consciously trying out a few ideas but the real sort of building up intuition which is the most important things is somehow done on a slightly lower cognitive level and I feel like I need to give enough space for that somehow by going for a walk and drinking water and clearing my head slightly to try and process some of the things that I've encountered during the day.

BH: You talk about this moment when you flick the switch and everything's in place, like that must be an amazing feeling. That must be… and you've had quite a few of them for early on. What's that like?

JM: Yeah it's a big mixture of emotions. So one thing that's always slightly depressing with maths is that you spend… ninety-nine percent of your time really not understand what's going on at all, and then suddenly you have the lightbulb moment but then you completely understand what's going on so it doesn't seem complicated at all. So, I look at other people's work and I think, oh my goodness, they're so clever, they're doing all these really complicated things that I don't understand at all and all of my work is super trivial. It's really easy because I understand exactly what's going on and it's really clear. And… this is obviously just the sort of… well I hope it's just the different perspectives that you naturally have but, yeah, that's one funny thing with maths that somehow everything that you understand is easy and everything you don't understand is impossibly complicated and there's no in-between ground at all. Similarly if you have proven a result that sort of you know other people have tried hard to prove and things and is a cute result in itself it's really rewarding when you feel like you have this… eureka breakthrough moment and so I tend to get this big adrenaline rush and I get very excited but I also get pretty scared. So after this like wave of adrenaline and euphoria I also have this big fear that I'm about to… claim a big mathematical proof which has some really obvious flaw in it and I'm gonna destroy my reputation in the mathematical community. And so I start gettin' very worried that maybe I've made some really idiotic mistake.

BH: How does one avoid making the idiotic mistake, you just… you just go through it again? Go through it with some collaborators and check your work?

JM: Yeah so it's a very important mathematical skill that you gradually develop about being skeptical about your own work and having an intuitive idea of what feels right and what doesn't feel right when you try and do proofs. So, when you're like very new to a problem often your intuition isn't too well developed and so it's much easier to make small mistakes and things. But this mathematical intuition I find is a very good… guiding… process as to whether I'm on the right track or whether I'm not on the right track and again this is just formed through trial and error. But it's also a general mathematical skill about being skeptical of your own work and being able to look at it critically and be like, hang on, have I really proven this… or, am I missing something here? And that's something that's often quite difficult to development but is a really important mathematical skill to develop.

BH: Do you ever worry that like the tap will switch off? At a time where it seems like you're going through such a fertile period of ideas and proofs and that, do you worry about it switching off or does it feel like it's just… it will continue forever?

JM: In the back of my mind there is a small fear about this, that, yeah, I guess at the moment I feel like I have many more problems to work on than I have time and I feel that there's so many different really interesting mathematical problems out there and you really don't need to be a genius to make… some decent progress on lots of mathematical problems that… if you put in the time and the effort and you learn the state of the art techniques and there's almost always some like small variations that can gradually inch forward the techniques. So I feel that… even if you lose a sort of creative streak or something then there should always be a… decent supply of problems out there…

BH: Like grunt work sort of?

JM: Well… but I think this grunt work is really important for the whole of mathematics so grunt work sounds maybe a little bit negative about it but it's easy to notice like, individual breakthroughs or a few amazing people who come up with singular discoveries but the context for which those discoveries are always made are when the field as a whole has gradually developed… techniques and has understood very subtlety the limitations of those techniques and how far those techniques can go. And, so lots of the slightly glamorous work is still hugely important for mathematics and you wouldn't possibly be able to have the breakthroughs without this grunt work as you say being done, so I feel that there's also going to be lots of fertile problems where you can make good progress and understand techniques and push things forward but at the back of my mind there's always this fear of… what if suddenly the tap goes off? That at the moment as I say I feel like I've got more things to work on than I have time to do but… often if I'm working on a challenging problem I'll go three or four months with making absolutely zero progress. It won't be three or four months where I feel like, oh, I haven't proven the thing but I've understood the problem a lot better, I can go three or four months and feel like I've made no progress whatsoever on this. And so this can be a little bit disheartening if you're… certainly if you're not used to this. Certainly after a while there's a little bit of… is it… that this is just the natural process of research or is it that I've lost it? And so there's always a small fear in the back of my mind, but as I said there's so many interesting problems and so many different ways you can make progress and contribute to mathematics that I don't spend too long worrying about it.

BH: Can I come back to prime numbers for just a bit?

JM: Yeah, sure.

BH: You must always get asked what your favorite number is? What's your favorite number?

JM: Two.

BH: Really?

JM: Yeah so… obviously I have to choose a prime number.

BH: Yeah.

JM: And… but two is somehow special in all kinds of ways. That the reason we… don't choose one as a prime number is because if you did define one to… lots of people ask me why is one not a prime number, you know? It… it can't be divided by anything other than itself but then when you have lots of results about prime numbers you'd have to say, prime numbers apart from one, but it is the case that we nonetheless have all these results about prime numbers where it says prime numbers apart from two, 'cause two somehow is special and different to them. So…

BH: But two's also a spanner in the works of primes?

JM: Yeah. So one would be a huge spanner in the works. And most of the time things work okay with two but often two's a spanner in the works. And so you'll say well we have to ignore two 'cause somehow that's a badly behaved prime. And so because two's special in this way and it stands out, I think two would be my favorite number.

BH: A badly behaved prime?

JM: Yeah.

BH: That's a category I haven't heard before.

JM: [laughs]

BH: Are there any other primes besides one and two, that behave badly?

JM: Umm. So in some ways, they get better behaved the bigger they are.

BH: Yeah?

JM: So in many ways two's the worst. I think… the phrase that other people have used is two is the oddest prime.

BH: Right. [laughs]

JM: [laughs] But then, three would may be the next worst behaved and then five and seven and so on. And so two just by virtue of being the smallest prime number is often the worst behaved prime number.

BH: I've heard you give this answer before but I think I should ask you about it. Why is it important to research prime numbers? Like why's got a little bit extra important about it for us to understand these numbers so well?

JM: So primes are like the atoms of arithmetic, the key point about prime numbers is that every whole number can be broken up into prime numbers and so if you can understand the prime numbers well then you can understand the whole numbers well. So somehow to me the whole numbers are the most natural possible things in the universe, that just the numbers are one, two, three, that we use to count are the most fundamental objects in mathematics, and somehow there's all these questions about whole numbers when you're looking at them being multiplied together that boil down to just questions about the prime numbers and even though whole numbers maybe sound like the simplest questions in the world, they boil down to these questions about prime numbers that we somehow don't understand at all. So we have the most natural things in the world but they're build out of these objects somehow we don't understand. So there's lots of questions about whole numbers that turn into questions about prime numbers, so for example maybe one of the most famous questions ever about whole numbers was Fermat's Last Theorem, which is asking are there solutions of X to power N plus Y to power N equals Zed to the power N. And this is now a theorem due to Andrew Wiles but one of the like… very basically first steps in the theorem is to say that because prime numbers generate all whole numbers, if you wanna show that there's no solutions to X to the N plus Y to the N equals Zed to the N, it's sufficient to consider only the case when N is a prime number.

BH: Right.

JM: And it turns out that because prime numbers are special in many different ways, it's much easier to understand the situation when N is a prime number then when N is not a prime number. And so this is maybe the very first step in the very long and complicated proof of Andrew Wiles on how to prove that there's no solutions X to the N plus Y to the N equals Zed to the N, when N is bigger or equal to three.

BH: I didn't realize that. So, I know it's an epic two hundred page proof but one of the key foundations of it at the start is actually he's only doing it for powers that are prime?

JM: Yeah so, and again you have this slight issue with two comin' up that… when…

BH: Badly behave two!

JM: Yeah when the exponent is two then you do get solutions to X squared plus Y squared equals Zed squared…

BH: Yeah.

JM: 'Cause, of course that's just Pythagoras' equation.

BH: Yeah.

[gentle violin music]

BH: So of the various problems that are out there with prime numbers, Twin Prime Conjecture we've already spoken about, like that seems like it might be top of the heap or Riemann Hypothesis as well… well, can you run me through the three or four biggest problems in prime numbers that you would most love to see chalked off?

JM: Okay so I think the Riemann Hypothesis is clearly the most important problem on prime numbers and that's the problem that I would really love to see chalked off.

BH: Are you able to describe what the Riemann Hypothesis is with just your voice? 'Cause I've seen you do it, I think, before quite simply, way more simply that I've seen where people are drawing strips and graphs and how would you describe what the Riemann Hypothesis is, to your linguist parents maybe?

JM: Yeah, so I think if it as just a problem about counting prime numbers. So how many primes are there up to a million, there's a very natural guess as to approximately how many primes there should be and the question is how good is this guess. And… the Riemann Hypothesis just says that this guess is actually very accurate indeed and so if the Riemann Hypothesis is true, regardless of what number you chose whether it was a million or a billion or a gazillion than we would have… we would know that the number of primes up to a gazillion is this natural guess plus some very small error term which is basically as small as you could possibly hope it to be.

BH: Okay so Riemann Hypothesis top of the list.

JM: Yeah.

BH: What else we got that you'd like to see knocked off?

JM: Okay, so as I said the Twin Prime Conjecture is the on that sits specially with me and yeah I really like that because if you're interested in the distribution of primes somehow the Riemann Hypothesis is talking about the large scale distribution of primes, how many primes are there up to a gazillion? Whereas the Twin Prime Conjecture's talkin' about the small scale distribution of primes, how many primes are there are really close together? So in some ways they're like the two extremes of the most natural questions you'd ask about the distribution. So for me, those two are both the most two… famous problems but the most important problems as well. Then for other problems, it turns out that Goldbach's Conjecture is very closely related to the Twin Prime Conjecture. So Goldbach's Conjecture is probably one of the most famous problems in primes, but for me it's very likely that if you can solve the Twin Prime Conjecture you might well be able to solve Goldbach's Conjecture and so you kinda get two birds with one stone in this…

BH: They have two famous birds. If someone does that…

JM: Yeah, exactly.

BH: …they're gonna like a prize or something.

JM: Yeah, very much so.

BH: [laughs]

JM: But it sort of means that because I'm in love with the Twin Prime Conjecture, Goldbach's Conjecture isn't so important to me as it might be to… some other people… and as famous as it is. Maybe the next most important problem for me would be Prime Values of Polynomials, so are there infinitely many primes of the form N squared plus one. This is another one of these very simple… to state conjectures and the answer as with lots of these things is, oh it clearly should be the case that's the infinitely many primes of the form N squared plus one, how could it possibly not be, but if had techniques that could handle this then there's all kinds of interesting questions, again linked to solutions of integer equations and there's applications in group theory as well that would follow of questions about primes of the form N squared plus one or in general like Polynomials taken prime values. So that'd maybe be my number three question.

BH: The Twin Prime Conjecture always seems to me like, why is this so important? Like the Riemann Hypothesis and how many primes are there, seems like, okay I can see why that's a really important question. But like the Twin Prime Conjecture always just feels a bit like… oh how many little freakish moments are there where two of them were near each other, like, just like… how many coincidences are there? How many times will I walk down the street and be lucky and find a pound coin? Like Twin Primes… are nice, they tickle my brain as nice things, but they don't seem important. They just seem like, happy coincidences and yet there's such importance is placed on this problem. Can you help me understand why it's important to understand these… freaky moments where you have consecutive basically primes.

JM: Okay so there's two reasons in my head. The first reason is that the primes are an interesting sequence and if you can't get really good specific mathematical control over the sequence, you maybe ask statistical question. Like how many primes are there? That's the Riemann Hypothesis. But also in general if you want to understand a sequence as well as, as I said, looking at these large scale statistics, you also often get a lot of information about small scale statistics. So how well they're distributed in short intervals and how often they come close together and things like that and so if you can prove the Twin Prime Conjecture, then it's likely that'd you know roughly how many primes there are that differ by two but also how many different by four, how many differ by six, and the question, although the Twin Prime Conjecture is state as just are there infinitely many times you get these lucky coincidences, it's quite likely that sort of the real

Twin Prime Conjecture is about how often do you have gaps of a certain size and it's all about a statistical understanding when primes come close to each other. So it's just the same as the Riemann Hypothesis but looking at small scale statistics rather than big scale statistics and that's what sort of statisticians might do for any other sequence that comes out of physics or nature or something. The second reason that I… so that's why I view it as a natural problem in and of itself, if you accept that the primes are interesting. The second things is that… [pause] it mixes multiplication and addition and primes are somehow naturally multiplicative objects because they're defined by every number can be multiplied by primes put together, but then you have this plus two feature that means that there's this slightly mathematically jarring interaction between addition and multiplication. And lots of our ideas behind it are trying to understand how multiplication and addition interact with one another. And so for mathematicians the sort of technical side of things, I think the Twin Prime Conjecture is fascinating because it's really trying to understand on a… it's a sort of toy case of trying to understand a much bigger problem of how multiplication and addition interact with one another and there's lots of very different subtle questions about how… this comes about and if you could prove the Twin Prime Conjecture you've likely made a big advance in how you can understand interaction between multiplication and addition which are the two most basic techniques in mathematics, operations.

BH: When I think of prime numbers I think of them as like individuals, seven and thirteen or if I see a big number I'm like, oh is that prime? You know, I often will get a big number and just put it into an internet search to find out if its prime 'cause I'm weird like that. But it seems like you're not particularly interested in prime numbers anymore as… individuals. To you they're just always P. Take you know, P one, P eight, like… like… do you feel like you've moved n from prime numbers as personalities, like it's almost like you watch football and you don't see like Wayne Rooney and the players you just see like, footballers.

JM: Yeah, this is certainly a bit like it that primes are… we know that primes are these really complicated numbers and so trying to understand them on an individual level is just a hopelessly difficult task and so we have to take this sort of more zoned out… blurred statistical way of trying to understand them 'cause this is the only way we have hope of getting any sensible answers. And so… these sort of overall patterns of prime numbers are definitely the sorts of things that interest me and not any individual… number itself where you write it down explicitly.

BH: So when they announce a new prime number. A new massive prime number's been found, you're a bit like… meh.

JM: Yeah. It's… it has… some interest and it shows sort of the computational… how far computational techniques and things can go and sometimes the ways that the algorithms that they use to find the prime numbers are… saying something interesting about the prime numbers themselves, but I definitely view prime numbers as some huge family and I'm only interested in properties of the whole family rather than properties of any one individual in the family.

BH: So if you know you write down someone's phone number or you see a number somewhere you don't, in the back of your head think, hmm I wonder if it's prime… like?

JM: Yeah I don't really do this. I definitely somehow notice numbers a lot more than I think a normal member of the public so… I remember there was a funny experience that my girlfriend was showing me her old textbook from undergraduate studies where… I think it was maybe a biology textbook, I can't remember exactly, and she was just flicked to a random page and was sort showing me some of the things that she'd learnt during her undergraduate studies and I looked at one of the diagrams and the first thing I did was I zoned in on the numbers because I was trying to work out, yeah I think it was some picture of a cell or something and so it was very small and I noticed that two of the numbers were inconsistent with one another.

BH: Hmm.

JM: And so I pointed this out and she found it completely bizarre that the first thing she did was open a random page of her textbook to explain some of the things we were doing and because I just zoned in on the numbers I noticed a typo on this textbook that's famous and is on it's fifth edition or something.

BH: [laughs]

JM: Just because the numbers didn't quite match up. So I definitely have something slightly odd about noticing numbers but… I don't tend to test an individual number for whether it's a prime number or not. I'm normally much more interested in the sort of vague properties of numbers rather than the individual properties of individual numbers.

BH: Because you've had a few like headline grabbing proofs and breakthroughs, do you find everyone asks you what you're working on at the moment? What's next?

JM: Actually they don't do that as much as you think, so I certainly get asked a little bit… oh what are you working on at the moment but I think this is normal for mathematician that it's… sort of maybe British people talk about the standard opening line is what's the weather? Maybe in mathematicians are like, oh what're you workin' on at the moment?

BH: Yeah.

JM: Because you get a good feeling for what other people are working on. So beyond that I find that people are… always interested in what everyone else is working on and it's always good… I find because I'm still relatively new to the subject and junior overall, it's good to see how other people pick their problems and how they choose what to focus on and where they put their focuses but I don't think I get I asked, what am I working on much more than any other mathematician, really.

BH: What are you working on at the moment?

JM: So at the moment I'm working on a result about… primes in arithmetic progressions. So, I've got paper that I've… been… writing up slowly over the past month or so, so that's almost finished now, so that's almost ready to go.

[gentle music fades in]

BH: Oh, well, I know what our next video's gonna be about.

JM: [laughs]

[music fades up]

BH: [laughs]

[music continues]

BH: That's all for today. I'll have links to some of James' work and his videos on Numberphile in the show notes. [music continues] The Numberphile podcast is made possible by the Mathematical Sciences Research Institute and this episode was supported by the audio engineering company, Meyer Sound. Again, links in the show notes. [music continues] You can support the Numberphile project on Patreon, go to patreon.com/numberphile, I'm Brady Haran, we'll be back again soon with another mathematician and another episode.

[music fades up and out]