Numberphile Podcast Transcript
Episode: The C-Word - talking Calculus with Steven Strogatz
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We record at the Royal Society in London, with Isaac Newton's original calculus papers.

Infinite Powers by Steven Strogatz

Steve's author page on Amazon

Some bonus video footage we filmed at the Royal Society

Hannah Fry with the same documents on Objectivity

And the Principia on Objectivity

The Royal Society

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[gentle piano music]

Brady Haran [BH]: Today's guest is Steven Strogatz. He's a mathematician at Cornell University.

[music continues with bells chiming]

BH: But Steve's also pretty widely known as a math popularizer. One of those people who pops up everywhere. He's in the New York Times, on Radiolab, places like that. He was recently in London promoting a new book he's written called Infinite Powers. [music continues] It's all about calculus. Now any book about calculus is obviously gonna feature Isaac Newton, so I arranged to meet

Steve somewhere special and as you're about to hear I had a few surprises up my sleeve.

[music fades out]

Steven Strogatz [SS]: Well, I've just arrived at the Royal Society of London, after walking from my hotel. We're now in a room that has portraits of many luminaries from across the centuries who were either Fellows of the Royal Society or Presidents of the Royal Society. The room is itself called, you please remind me… the Chamber? The Council Room?

BH: It's the Council Chamber. This is where the current bosses of the Royal Society have their meetings, so like the president and the vice-presidents all sit here and do business, so you're actually sitting… you're sitting I think one chair down from where the president sits.

SS: [laughs]

BH: So you're in a vice-presidential chair.

SS: [laughs] Oh, okay. I'll take it!

BH: And over your right shoulder I see Charles Darwin peering at me.

SS: Peering.

BH: And over your left shoulder we have Humphrey Davy. So we're in a, you know… we're steeped in science history but because you've got a book out at the moment about calculus, Keith Moore, the head librarian, has dug out some treasures especially for today. And look I'm gonna let everyone in on a bit of a secret, Steve was crying a few minutes ago.

SS: [laughs]

BH: Like literally crying. He has tissue out dabbing…

SS: Don't start me again!

BH: [laughs]

SS: I've been told that I'll drop… I'm not allowed to drop any tears on the original manuscript of… De Analysi Newton's first write up of calculus meant for a broader consumption. I mean he had plenty of manuscript for himself but De Analysi is his attempt to write something coherent for… still not for a wide audience but for his disciples. For his little circle in Cambridge, but still I was getting… so… agitated [laughs] overwhelmed to see his hand. Oh I will again, I'm sorry. [laughs]

BH: Why is that, Steve? Why… you know you're having just written a book about calculus you must be so steeped in all this stuff. What is it about… the documents like? 'Cause obviously you're now seeing the handwritten one in Isaac Newton's handwriting.

SS: [stutters] You know it's not some kind of deification of Newton. You might think it's that. It could be that. But I see… well probably it is then [laughs] something like that.

BH: [laughs]

SS: But you know the one that got me going here, just a minute ago, was Isaac Barrow who is often regarded as Newton's mentor to the extent that he ever had a mentor and Barrow was saying something about Newton to his… to a colleague named John Collins, but it's a line that I've written about. I've quoted it but to see Barrow's hand saying this… [sighs and laughs] I'm sorry just having so much trouble getting through this but you know he talks about his friend such extraordinary proficiency and genius and these things, these mathematical things. [pauses] Um… which is quite of understatement.

BH: Do you look at that and feel emotional because you're looking at like, you know, you have some understanding of these mathematicians talking about each other or because you've devoted, you know, a chunk… so much of your time to it?

SS: Well I think it's… this maybe the smartest person of all time that we're talking about here. I mean this is the world's greatest genius. Einstein is wonderful. Archimedes is wonderful. But in terms of changing the world, all the Hitlers and all the Caesars and these petty generals will be forgotten long after Isaac Newton is remembered. I really do think so. I mean this is the person who made the modern world possible. Not single handedly, as he said he built on the shoulders or stood on the shoulders of giants but it's just astounding what Newton accomplished in calculus and physics and astronomy and optics, you know, and in fields that we don't even take seriously anymore. Alchemy, which led to chemistry. Chronology of the Bible. Not something that scientists normally think about but Newton was most serious about that probably more than anything. He hoped to be remembered for his work on the Bible. Anyway, so that's not what gets me emotional [chuckles], his Bible work, but still…

BH: What does it… seeing it in real life like see the actual thing what's strike you most about it, I know…

SS: Well there's the tiny handwriting. [chuckles]

BH: Yeah.

SS: And so immaculate. And for some reason I'm moved by the cross outs. That here's… that you think of him as superhuman but he's not, he makes mistakes and he crosses things out just like the rest of us. Well, not maybe just like the rest of us but still. It is a person that we're looking at and he was quite young. I mean we're seeing someone who's in his early twenties here. I'm also somewhat moved by his psychological difficulties. This is a person who is so sensitive and secretive and afraid to be criticized, you know, I see him writing, but who is he writing for? He's not really writing for publication. Something about his fear moves me. You know, what is he afraid of? But, you know, going back to his own story, for people who don't remember or don't know, he was born without a father. His father died while he was in utero and so he was born fatherless and his mother, you know, it wasn't easy to be a mother with no husband and so she tried to get remarried and did. She married a much older man who was a reverend in her town and the reverend didn't want new son. He wanted a new wife. And so Isaac was… a burden to his mother and was sent off to live with his grandparents and he felt that he was abandoned by his mother. And later in life when he was nineteen he wrote about sins he had committed up til that point in his life and one of them was that he wished that… his mother and his step-father would die. And he also wished that he would die… himself. So, you know, this is a very troubled lonesome little boy who grows up to be an extremely troubled young man who then goes on to be this towering intellect. I see all of that when I see his hand and his secrecy, and you know, writing this book for nobody. You know it's a strange thing. It's a book for himself and maybe to be shared with a few people but I think even there he was uncomfortable doing that.

BH: So when you look at something Isaac Newton has written and you're projecting a lot of these things on to the piece of…

SS: Yes I am. [laughs]

BH: Is it… are you look at it through the eyes of someone who is like… this exceptional exceptional case, which clearly he is in so many ways. Or is it part of looking at the mathematician in you and remembering when you were in your early twenties and thinking…

SS: [laughs]

BH: Oh, am I rubbish? And are people gonna laugh at my papers?

SS: [laughs]

BH: Like you talk about that secrecy and that fear that you kind of…

SS: Yes.

BH: …sympathize with. Are you looking at that in an everyday and remembering what you were like as a young mathematician? Or are you thinking ah, you know…

SS: Yeah I don't think I'm making any kind of implicit comparison there.

BH: Right.

SS: It's I think I feel a sense of awe that for other people might be religious awe, you know, in the presences of a deity. That's why maybe earlier I inadvertently or maybe subconsciously came out with this word deify. You know that…

BH: Yeah.

SS: Am I deifying him? I probably am in some way that for me as a non-religious person maybe there is some version of reverence happening here and so the same way that a worshipper of some other more traditional religious figure would feel seeing the Shroud of Turin or something like that. I think I'm having some of that right now.

BH: You're fanboying.

SS: I'm fanboying but…

BH: Yeah. Yeah. [laughs]

SS: It's a little more cosmic that that. [laughs] A little more reverential than that. But yes it's something like that.

BH: Newton did so many things and you know you could talk about things Newton for a long time. Let's stick with calculus 'cause that's all the documents we've got out in front of us.

SS: Mmm. Yes.

BH: And what's been on your mind a lot lately. How would you describe… 'cause calculus is such a famous word, right?

SS: Mhm.

BH: But I think a lot of people don't know what it is…

SS: Mhm.

BH: What it is even at the most basic form. They just know it's something that smart people can do. How would you describe what calculus is to someone in the pub who doesn't even remember studying mathematics at school?

SS: [laughs]

BH: They're just like, oh you've written a book about calculus. What is calculus? I've always wondered what calculus is.

SS: Yes.

BH: I always hear that word but I dunno what it is.

SS: Sure. Good question. Yeah that's a challenge. I like visualizing my [laughs] this anonymous character in the pub. Right so if the person is willing to listen to a discussion about mathematics I guess I would first say that mathematics and calculus specifically have been used to give us the modern world. So, not on it's own, I mean certainly calculus has had support from fields like engineering and physics and technology. All of that is necessary to create things like our cellphones today or microwave ovens or the GPS gadgets that help us find our way home. But calculus is actually built into all those things too. They're not pure technology. They are technology that use mathematics relating to questions of how things change. So if I had to give it to you in one sentence or even one word, calculus is the mathematics we use to describe things that change. Specifically things that change continuously, or smoothly. Not in jumps, not it jerks but things that change gradually and smoothly. They could be quite rapid but so what sort's of things? Well anything that's moving, is changing it's position, so when we study how to get a person on the moon, you know, the mathematics of rocket science is calculus. If we want to understand how the amount of virus in the body who has HIV decreases as they're given triple combination therapy to keep their disease at bay, studying the dynamics of a viral load, the changing levels of virus or the T-cells in their immune system, calculus is used for that too and has helped us create those therapies that I just mentioned. Calculus is the mathematics of change and until it was invented, mathematics was sort of a static motionless inert thing. Which is sometimes what you want. I mean you don't want a building to move. You don't want a bridge to shake or fall down. So sometimes you're happy with things at equilibrium, at rest, unchanging, and the mathematics before calculus was all about a world at rest.

BH: So Steve, before calculus came along how did people deal with things that were changing? 'Cause obviously they still had things moving. There were still like… presumably there were still horses and things and…

SS: [laughs]

BH: Things still grew… like you know…

SS: That's right.

BH: So there was movement in the world. Was that just something that was completely untouchable for mathematicians?

SS: Yes. Essentially it was. It was… something that was as plain as, you know, the fact that we all get older, or that children grow up. I mean everyone knows that things change. We see leaves falling, we see water moving in a stream. But mathematicians and scientists couldn't really describe it. Not very well. There was only one case that could be described which was the simplest kind of change which was change at a steady rate. So if something changes at a constant rate meaning like, you know, if I give you… if I say one banana costs… I don't know what would a banana cost? A dollar. Or a pound or something. I dunno that's a pretty expensive banana but [laughs] you know how much will it cost if I change the number to ten and then you say ten time one will be ten dollars or pounds. We've done a kind of mathematics of change but it was something so simple but so something like a ball dropping off a tower… you know or I stand on a very high building as Galileo in the legend was said to have done and then he dropped the ball. Just to understand how things behaved when they fall was a mystery to science forever. Aristotle had theories of the motion of a falling object and… his theory was wrong. Partly because we couldn't measure time very accurately. We did't have video cameras, you couldn't track the motion of a dropped cannon ball or a pebble. So it took thousands of years to even… I mean it sounds like a boring question, something dropping. It is a boring question. But even that was beyond science for most of humanity's history. And so it's only, you know, when you start something like the 1600s that we begin to understand the very simplest kinds of motion through calculus and science.

BH: What happened in the world? What changed? Or what was the catalyst for calculus to be… devised? Was it just that serendipitously a few smart guys happened to exist…

SS: [laughs]

BH: …or was there some problem that had arisen that sort of, you know, the necessity that was the mother of this invention?

SS: Mhm.

BH: What happened at this time in history?

SS: Well yeah so there were definitely problems that we outstanding, that needed solving and the mathematics arouse in response to those challenges. I started the story here in the middle by talking about motion. The real truth is that calculus began almost two thousand years earlier, and the questions there were problems about shapes that were curved. Like a circle or a sphere. Something that was made up of straight lines but that had curved boundary. Even a question as simple as how much area would be contained in a circular field? We couldn't answer that question. We can answer a question about the area inside a rectangular plot of land, and that was a question that would arise when tax collectors needed to know how much should they tax a person for their field. It depends how big the field is and that depends on it's area, which is the length times the width. Fine that works well for a rectangle but… no I don't wanna give a [chuckles] a confusing pseudo-history here, it's not that we needed to tax circular fields.

BH: Yeah.

SS: I think it's more a typical case of human curiosity. That the circle was a natural shape. People see circles when they look each other in the eye. They see the circle iris and pupil. We see the moon as the shape of the circle. We see circles in the ripples on a pond. So we've been aware of circles forever and they have very important symbolic significance to us. We wear circular wedding rings, we think of the cycle of the seasons, and eternal, you know, life, and so on. So anyway, there's this big emotional dimension to circles, yet we can't answer the simplest thing about a circle. Like how far is it around the circle, the circumference? Or how much area is inside a circle? And so that was the original beginning of calculus, was answering questions like that about simple curved shapes like circles and spheres. And there was no way to do. You won't find the answer for the [laughs] believe it not the area of the circle does not occur in Euclid's Elements. There is no formula pi R squared that we all learn in high school where R is the radius of the circle and pi is that mysterious number that, you know, has infinitely many digits that never repeat and show no pattern. That's already a clue that there's something deep happening with circles. That pi is so intimately connected with them. And so the first person to really solve the riddle of area or circumference of circle was Archimedes, who is a genius on the same scale as Newton. And so the secret to solving questions like that was the strategic use of infinity. This amazing, scary for many people, concept of something that never ends. Archimedes showed that if he imagined chopping up a circle, say for the area question, if he chopped up the circle into infinitely many pizza slices, infinitesimally thin, he could rearrange them to make a rectangle and then find the area that way. And so it was magnificent leap of imagination to see a circle as equivalent to a rectangle through the bridge of infinitely and infinitesimals. But that's his accomplishment and you know making it very careful and rigorous but that's essentially the big new idea is that you could use infinity to solve questions about curved shapes. And so back to your question, what was it that gave rise to calculus in the time of Newton? You know this outgrowth of Archimedes' work thousands of years earlier. It was a similar question. Instead of now analyzing curved shapes, which was still actually a live question for Newton, there was a new question which was analyzing… motion that changed it's character as it proceeded. So something that's sped up or slowed down. That accelerated. That was the kind of question that arouse in thinking about the motion of the planets. The motion of falling bodies that Galileo had worried about, Kepler's Laws, The Copernican system of the planets going around the sun. All of those raised questions about variable motion. Although Galileo and Kepler had laws, they couldn't explain them. And so that was really the big million dollar question for Newton was could you explain what Galileo and Kepler had found? And of course he did and put their two bodies of work together into this magnificent synthesis, the Principia, that is the beginning of the scientific revolution.

BH: Oh, I just figured out a way I can make you cry again.

SS: Don't do it! [laughs]

BH: I'm gonna get Keith… I'm gonna get Keith to go and get the hand written Principia from downstairs. [laughs]

SS: No, no, no! [laughs]

BH: That'll really set you off.

SS: Yes it will! [laughs]

BH: So, anyone with like a mild interest in the history of science and mathematics. Even if they're not an expert on calculus probably knows that there's this famous stoush… this famous beef… was it Newton or was it his guy called Leibniz, who invented calculus?

SS: Yes, yes.

BH: Because they both came up with things at the same time. After all the work had been published and the mathematical dust had settled there was almost like an investigation into who…

SS: Hmm.

BH: Into who invented it. And in effect this pile of papers sitting here to your right is the Royal Society's like evidence bag. This is all the papers that Newton and Leibniz and everyone wrote that they put together to try and figure who said what first. So we're almost seeing the leftovers of the trial. This is something you've obviously looked into in writing a book about calculus. Can you sort of summarize what happened separately with these two people and then can you give us the verdict as to who the winner is?

SS: [laughs]

BH: [chuckles]

SS: Well, yes, alright so these are the facts. Newton invented his version of calculus around the middle 1600s. You could date it 1665 or '66, but it's around then. Leibniz is definitely later, about ten years later. He first becomes interested in advanced mathematics at around the time of the 1672 when he's in Paris, meeting with Christiaan Huygens, who is the greatest mathematician on the continent. And it's a measure of Leibniz' genius that starting from as a beginner within just a matter of four years he is almost at Newton's rank. And he's not even particularly mainly interested in mathematics. He's also a diplomat. He's a logician. He's a linguist. I mean he's an extremely versatile genius himself. Maybe the most versatile in a century of geniuses and that includes Descartes and Galileo and Bach and you know, so Leibniz, he's got game, you might say. [laughs] okay?

BH: Yeah, okay.

SS: I mean he can do everything. I mean what's confusing is that he was shown some of Newton's manuscripts. There were these intermediaries. There were people who were… John Collins, it's wonderful that, you know, we're in the Royal Society building, so the secretary of the Royal Society at the time, Henry Oldenburg, served as an intermediary when Leibniz would write to Newton, he wouldn't write to Newton he would write to Mr. Oldenburg who would then manage to get the letters to Newton. So we have inquiries and in this book that sits to my right I have the letters [laughs]

BH: Yeah.

SS: In the hand of Leibniz and in the hand of Newton.

BH: Yeah.

SS: Back and forth with Leibniz' questions to Newton and then Newton's responses and so… [sighs] there was always a question of did Leibniz somehow plagiarize Newton? Was he given clues from seeing these manuscripts?

BH: Yeah.

SS: And then somehow incorporated those clues into his own version of calculus.

BH: Hmm.

SS: So I'm not a historian. I should make that clear, I don't really totally know what I'm talking about but what from I can see I think it's clear that Leibniz essentially independently thought of calculus. His version of it is quite different.

Notationally certainly. And conceptually. It's also clearly later and it's also clearly published first.

BH: Right.

SS: So that's the interesting thing. Leibniz definitely publishes before Newton because as I say Newton's very secretive.

BH: But he is showing friends like… like it's amongst…

SS: That's right.

BH: He just hasn't published in… well I guess they didn't really have journals then did they?

SS: No, they didn't have journals.

BH: Yeah. So what did published mean in those days?

SS: You could publish a book. You could send a book to a publisher and it would be published.

BH: Right.

SS: And so Leibniz does publish his version of calculus and certainly he has disciples who publish the first text books on calculus.

BH: Right.

SS: So this has always been the argument. Who's first? I mean if you measure it by publication you have to say Leibniz is first. But who was first conceptually? There's no question that Newton is first. And also to my reading there's no question that Newton is the superior mathematician. He just goes much deeper and you see this in some of the letters and I would be curious to talk to our librarian friend Keith about this. Because one of the letters has Leibniz trying to show that he has credibility and he offers an infinite series to Newton. It's a particular kind of calculation in calculus, use of infinitely many algebraic terms or numbers sometimes. Anyway he has a particularly impressive, he thinks, infinite series to show Newton that he's to be taken seriously. And Newton writes back a long the lines of, you know, you can't imagine how much pleasure it has given me to see this infinite series and then I don't have his… a paraphrasing, a gist of it is considering that I already know three other ways of doing it myself…

BH: Right.

SS: I never expected to see a fourth way.

BH: Okay. [laughs]

SS: [laughs]

BH: Right, yeah.

SS: You know this is supposed to be a compliment but it's really a very sarcastic insult. Like why are you bothering to tell me something I already know how to do three other ways?

BH: It's like if I showed you my Olympic Gold Medal and yeah said oh I've not seen one like that. It's nothing like eight I've got at home.

SS: [laughs] Exactly!

BH: Yeah, right, right.

SS: That's exactly what Newton says to him. And then in response he sends him a letter that is filled with extremely arcane difficult fruits of the calculus that he has developed and Leibniz doesn't understand where these come from. Which demonstrates that his calculus is not up to snuff compared to Newton's. And he even asks questions and Newton then in another letter gives him a very pedagogical, almost condescending, explanation. As you would… like lecturing a child.

BH: Okay.

SS: So… okay I mean I think I've answered it. But as far this document that I've got in front of me, when Newton was president of the Royal Society he… convened I believe this is the correct history or someone convened a priority dispute. Like as you say, a trial.

BH: Yes.

SS: You know, the documents to establish… well in the case of the Royal Society the hope was to establish that Newton was first.

BH: Yes, this is what this is.

SS: This is what you're flipping in front of me.

BH: Yeah these are… this is, as I said, this is the evidence that was used to then… and then conclusions were drawn by the Royal Society.

SS: Yes.

BH: Unsurprisingly the conclusion that being that our man Newton was the man.

SS: And for a long time it wasn't known because I think it wasn't revealed that Newton himself was the judge and jury.

BH: Yes. [laughs]

SS: Of this, and wrote much of the analysis. And he was… it's very… which is to say I mean he was deceptive about it. He didn't sign his name to things that he wrote.

BH: Yeah.

SS: He made it seem like other people had come to these conclusions, it was really him.

BH: Steve, you said that, I mean besides the fact that Newton and Leibniz used sort of different notation. Was deep under the hood what they were doing the same? Or were they cracking the nut in completely different ways?

SS: It's the same. We see the equivalence of what they're doing.

BH: Okay.

SS: That's definitely the case here. We learn both methods today. But Newton's method is more physical. He's thinking about things flowing. In fact he doesn't use the word calculus. He refers to his subject as fluxions, the study of things in flux. And for him functions are… he calls them fluents. A fluent quantity. Something that flows. He's very much thinking about time. Leibniz does not think about time. He's thinking more about discrete objects. He… is very visual and geometrical but he comes at calculus by thinking about discrete things that become more and more nearly continuous. But he doesn't… time is not nearly as important to him, nor is physics. It's an outgrowth of geometry for him. I mean Newton is very geometrical too but… yeah they look at it differently. It turns out that today when we teach calculus we're teaching in the Leibnizian framework. His notation is so elegant. The integral sign that we all know is first written down by Leibniz. And his D by DX, all of that is Leibniz. It's just a much better notation. Newton has a funny notation with dots over the letters and so to do a fourth derivative he would have to write Y with four dots over it. We don't do it that way anymore.

BH: So I'm aware that Leibniz' notation is what stood the test of time. So when I learned calculus in high school, which is a…

SS: [chuckles]

BH: …I have to admit is a bit of a distant memory but I did learn calculus in high school. Besides using Leibniz' notation was I also learning Leibniz' methods or was I doing it more in the Newton way? Or is that…

SS: I think they are more or less indistinguishable at the level of the calculus we learn today.

BH: Okay.

SS: So both of them… I mean the reason that they're often called the creators of calculus despite Archimedes having many of the ideas two thousand years earlier is that both of them saw the connection between the two big halves of the subject. So we have derivatives and integrals or we speak of differential calculus and integral calculus which roughly speaking means chopping things into infinitely… many small parts that's differential calculus and putting them back together, integrating them into a whole is integral calculus. And you need both to do the analysis in the style that I say Archimedes pioneered of breaking hard problems into all tiny, you know, many many tiny pieces that are simpler, solve those put them back together. That's the heart of calculus, that idea. The infinity principle. Nobody really calls it that but I would say that is the methodological heart of the subject, that you break anything that's continuous, whether it's a shape, a continuous curve, a motion, anything changing smoothly can be broken into infinitely, infinitesimally small parts that then we can think of as being constant or somehow changing at a constant rate, or being a straight line, that's the great bridge to the existing mathematics of straight lines and constant speeds and so on. Anyway both of them have that idea, both Leibniz and Newton and they both see the connection between the two halves of calculus. The putting together and the taking apart through something we now call the fundamental theorem of calculus. So since both of them, I mean hesitate to say discover it, because historians will point out there were also precursors of them, people like Isaac Barrow and James Gregory, but okay roughly speaking Newton and Leibniz make calculus systematic. They put all the different techniques that are lying around together into unified theories that are very close to the theories we still do today. I mean they've been improved over the years by disciples of both but… yeah, they're the progenitors of modern calculus. And we learn from both of them today.

BH: Do professional mathematicians, like someone at your level who's at, you know, the upper levels of mathematics, still use calculus on a day to day? Is it still a useful tool or is more for the beginners and you use whole new methods now and calculus is almost had it's day?

SS: It is the beginning. It's the first course for us in college. We don't use it very much. When I speak of calculus nowadays I'm thinking of it and all its outgrowths. Everything that spun off from calculus, but that would be unconventional terminology. We would normally speak of differential equations, and then there are subcategories of ordinary ones, and partial differential equations. You know that's a distinction whether we're concerned with things that only change in time let's say or in both time and space simultaneously. We also concern ourself with waves and putting waves together to make, you know, well say if you're looking at the question of how do you compress all the songs that fit on your phone? Data compression uses techniques call wavelets that grew out of Fourier Analysis of sine waves representing complicated functions as sums of sine waves. And so to me those are all part of calculus, but nobody else would call it that, you know, we think of wavelets and Fourier Analysis as a separate subject. I suppose the modern term should be continuous mathematics, which includes calculus as the first instance but then spins off in, I dunno how many, hundreds of directions. And it is the dominant framework for mathematics today, except for those fields that are very oriented towards computation. So anyone using computers has to learn discrete mathematics and linear algebra has become very important. So it's not all calculus all the time. I mean what's been great in the hundreds of years after calculus is that other older branches of mathematics were reinvigorated by fusion with calculus so. A subject like number theory which goes back to ancient times even to the Mesopotamians, you know, thousands of years before Christ. When we combine it with calculus we get modern subjects like analytic number theory that give us things like the Riemann Hypothesis, the greatest unsolved problem in mathematics today which by the way has been wonderfully covered on earlier episodes of Numberphile.

[gentle piano music plays]

BH: You were talking before about that kind of momentary human connection you feel with Newton when you look at these papers here.

SS: Mhm.

BH: Do you wish you could go back and tell him what came next?

SS: [laughs]

BH: 'Cause you talk with such enthusiasm about the mathematics that spun out of calculus.

SS: Yes.

BH: That we use today, I mean… you could blow his mind, couldn't you? If you could go back and…

SS: I think so. It is an interesting thought experiment. I tend to think more of Archimedes, what would he feel if he saw computers and the mathematics that grew… but certainly Newton himself would have been astounded by what we can do. It's interesting though, you know, I mean in terms of say computers there are in these pages in front of me we can find calculations where Newton is computing logarithms to fifty digits, by hand. He's just carefully writing out row after row of fifty digit numbers and adding them together and occasionally crossing out little mistakes. This is an unusual mind who wants to write fifty digits and is… and he even says in one place maybe we can find in these pages that he's… I mean again paraphrasing, he's embarrassed by what he has done because he had… he says something like I took all together too much delight in these inventions for having no other business at that time, you know, like he had nothing better to do so he just goes and calculates everything. But anyway, no, it would be wonderful to show what we could do today. I don't… I wonder whether he would have a generous spirit in reaction to it. You know, if he knew how Einstein had superseded him in physics… how mathematicians have taken what he's done and gone so much farther. I think he would. I think he was. Certainly he was very respectful of what Greeks had accomplished and Descartes and, you know, would he be a grouch or would he be a good sport about it?

BH: I think you're being generous from everything I've heard or read about Newton. [laughs]

SS: [laughs] Well maybe so, maybe so, maybe so… [laughs]

BH: I went to a talk that you gave recently in New York, and one of the parts of the talk that I found most interesting and this just probably just says more about me than anything, was when you were talking about the birth of your current book. You were talking about working with the publisher through it. How to write it? How to divvy up the chapters? What do call it? And things like that. I find that really interesting because it sounds like that was even though you've already written previous books quite successfully. It sounds like it was quite an education for you that kind of to and fro you have with the publisher. You saying this is what I think the book should be about and what it should be called and them saying [groans] maybe not.

SS: [laughs]

BH: Maybe not.

SS: Yes. It was an education. It was a very very positive experience. So my editor was… a gentleman named Amon Dolan who has become a very good friend of mine through this ordeal. [laughs]

BH: Right.

SS: Writing a book under his tutelage. I really think of it that way. I had never tried to write a book before that had a story from beginning to end. I mean I had written essays, in science and in mathematics we're used to writing papers.

BH: Hmm.

SS: That form is comfortable. I'm comfortable writing essays and my previous book, Joy of X, as a collection of chapters each about only fifteen hundred words or two thousand words, that appeared in the New York Times as very short pieces. You could read them in five or ten minutes.

BH: Right, so that was quite disparate, you know?

SS: Yeah.

BH: Yeah.

SS: And there was no arc to the story.

BH: Hmm.

SS: And there didn't need to be, it was just thirty chapters about interesting little topics.

BH: Yeah.

SS: I hoped. So that I knew how to write but how do you write and sustain a story about something as bulky and unwieldy and complicated as the story of calculus over its twenty-five year span and many continents. I didn't really know how to do it and suffered greatly just to produce a first draft. And when I did I was surprised, it wasn't the book I meant to write. It was much more about history than I thought… I'm an applied mathematician. I'm very interested in how calculus connects to the world around us today.

BH: Hmm.

SS: How it has changed the world in subjects like philosophy and politics so not just technology. And yet that wasn't the book that I produced in the first draft so the editor said to me that, you know, it's a lot of history of math and [chuckles] for people who like even those of us who are sort of interested in this that maybe we would like to hear more about how this is used today. I had one chapter that had an example of HIV as I mentioned earlier that HIV treatment has been informed by calculus analysis of dynamics of HIV levels in the body over time.

BH: Hmm.

SS: He said that was a very compelling example. If you could have one thing like that at least in each chapter, that would really… wouldn't that be interesting?

BH: Okay.

SS: You know the editors are gentle like that, the way they don't tell you… they might say, yeah, and I thought golly… god damn you're right! That would be… that's what I meant to do in the first place I need to do that but it required another half a year of work to figure out what belonged in each chapter of contemporary interest. Anyway it was a lot of work and you know at the end I thought what I had produced was a book that should be called the Language of the Universe.

BH: Right.

SS: And I was told that sounds like a science book that maybe only scientists would want to read. I dunno, I still like that title but… but Infinite Powers was my editors name for the book and I said I don't really even know what that means.

BH: Yeah? [laughs]

SS: And he said well everyone here in the marketing department…

BH: [laughs]

SS: …you know, thinks it sounds cool, and that's the point, maybe you don't know what it means but the people you want to read it [laughs]…

BH: Do you know what it means now?

SS: Uh… well I think it's supposed to be a play on my idea that infinity is such an important in the story of calculus.

BH: And it's powerful too.

SS: And it's powerful, but has it given us infinite powers? Well that's a bit of an exaggeration but it has given us a lot of power and there's a probably a play on the word power series because Newton uses powers of the variable X. X squared, X cubed and so, to take the use of infinity to a new level.

BH: 'Cause I find it interesting that you haven't used the C word in the title. You haven't got Calculus in the title.

SS: No it's not.

BH: Why do you think that is?

SS: [laughs]

BH: Do you think…

SS: I…

BH: Do you think it's a bad word? Like I feel like the word calculus is synonymous with smartness. The way that rocket science… ah rocket scientist or calculus it's like just shorthand for smart and complicated.

SS: Hmm.

BH: But unlike rocket science…

SS: Mhm.

BH: It's got like a bad rap to it.

SS: [laughs]

BH: And it's a word that carries baggage. It's almost become shorthand for too complicated to deal with.

SS: Yes. Well, a good point and for many people it's a very aversive experience to take the high school or freshman calculus course in college.

BH: Hmm.

SS: I can't tell you how many people. All math teachers have this experience. You meet someone and they say oh math, yeah I used to like math until I got to calculus.

BH: Right.

SS: So, it's probably not a selling word.

BH: Hmm.

SS: For a book aimed a general audience. I don't know if it's not. Maybe it would be. I mean I certainly don't feel shy about using it. I used it in another book. Calculus of Friendship.

BH: Hmm.

SS: So it could have been called that, but no I think Infinite Powers made also call to mind Marvel movies, you know, maybe this sounds like superheroes and maybe that’s kind of fun and…

BH: Yeah?

SS: I don’t know. And who knows, titling a book is not an easy job but… the response to this seems to be okay. Anyway you’re right though, it is a word that has come to be synonymous with complicated and difficult. And I think maybe it’s more informative to say what is at the heart of the subject which is that we’re trying to make complicated things simpler. That my editor for him this was a big a-ha moment. He said, I realize now after reading this calculus is not about complexity, it’s about simplicity. It’s about breaking problems into smaller parts to make them simpler. And it’s a systematic use of this idea except that it’s… a manic idea where you never stop. You keep cutting and cutting the problem until you infinitely often until it’s infinitesimally small. So it’s taking this idea of divide and conquer a frenzied limit. No pun intended. I mean we’re going all the way to infinity, that’s the big new idea.

BH: Is there anything not in the book? That you…

SS: Oh so much!

BH: That you would have liked in there or…

SS: Yes.

BH: Like what… give me the director’s cut.

SS: Breaks my heart.

BH: Yeah.

SS: Well, I found I couldn't get out of the 17^{th} century. I felt I needed to talk about Galileo and Kepler although it's not customary to have them in a book about calculus but because they were so powerfully motivating for Newton and then likewise Newton studied Descartes and learned a lot about how to combine algebra with geometry through the work of Descartes and Fermat so they had to be in there. And of course Newton and Leibniz. So I have 6 important people to discuss and they're all in the 1600s, and I want the book to be a 21^{st} century book. So, that's what I mean when I say it became too much about history of math and as a result partly the time was running out, I mean there are real world considerations. I needed to finish this and get back to the rest of my life and also the book was getting too long. I didn't get to cover some of the greatest mathematicians of all time who were very very important in the story. Gauss is not mentioned, Euler is not mentioned. There's no Lagrange, there's not Cauchy, you know, it's ridiculous what's left out of the book. [chuckles] It kills me!

BH: Yeah.

SS: And yet they're missing. On the other hand for the reader I have in mind who is this person in the pub that you're talking about, maybe it's enough? You know, I've given the heart of calculus and then after that it's… I have one character really Fourier sort of stands in for everyone after Newton. Not by choice, just that's the way it turned out. I thought if you had to pick one person who could summarize the three hundred years after Newton to the present, I think Fourier carries the most punch because his work on waves and how we use waves in everything from radar to microwave ovens to GPS and all the rest. Image… you know, face recognition all those sorts of things. I thought Fourier was at least get me some of the way there. And then I wanted to talk about very contemporary things like Artificial Intelligence and what that's, you know, may do for the calculus. Will calculus be obsolete as computer's become more powerful? So those are some of things that are touched on towards the end.

BH: Why would calculus become obsolete?

SS: Well… there are certain problems that we can solve with calculus. Once we have problems with too many variables, it's not that calculus can't help us. I'm thinking let's suppose we wanna analyze the orchestra of biochemical reactions, the symphony of chemical reactions in cell. You know, suppose we wanna solve cancer with the help of calculus. We know that certain genes go awry in cancer, that cells start dividing uncontrollably. What's going wrong? You know, what is it that's wrong in the cell at that point? Well, people in systems, biology and oncology are learning about all the different biochemical reactions in healthy cells and in cancerous cells, and so we have in mathematical terms problems that might involve thousands or tens of thousands or hundreds of thousands of variables. They're all described in their in their changing behavior by differential equations, so calculus would still apply but we can't visualize what happens in a space of a thousand dimensions. You know, that is we used to Descartes with his X and Y axes, were good at picture in two dimensions, and three. But a thousand dimensions we can't picture and so a lot of what we use calculus for is limited by our neural hardware… our wetware in our brains. We're just not up to it. But computers don't care about a thousand dimensions or a hundred thousand. They can calculate in any number of dimensions with any number of variables, so that's where I foresee calculus going. Trying to understand the behavior of complex systems that are that complicated with the help of computers. I don't think we're gonna be able to keep up. I think it will… it may be that computers will solve cancer and have to tell us the answer and we'll have to accept it. But we won't understand but it will turn out to be right.

[gentle violin music]

BH: These papers here that we've been looking at, you know, this is when calculus was brand new, this was like shiny new thing that was blowing people's minds. And then it developed and, you know, the field of mathematics developed, you talked about this continuous mathematics and things like that. Does sort of this calculus continuous mathematics still have like a bleeding edge research frontier, mathematically? Like is it… are there still new things being done or is this like a nut that's been cracked, and mathematics has moved on to other things? Or are there people still making new calculus discoveries?

SS: Absolutely. There are… well the subject has grown into what we would now call analysis. So no one would speak… I mean what I keep calling calculus or continuous mathematics, the rest of the mathematical world calls analysis. And so we have real analysis dealing with the real numbers, complex analysis with the complex numbers, you know involving imaginary numbers and so on and then as I say differential equations, partial differential equations for problems in space and time, both. We have dynamical systems and chaos theory, all of these things are still very active fields and every year… or sorry it's every four years we award Fields Medals, you know, to the greatest mathematicians, young mathematicians, and you can check, calculus is very well represented among the Fields Medals every time. So it's certainly at the cutting edge still, those as I say no one would call it calculus. We would speak of dynamical systems or statistical mechanics or something. Still what are those big conceptual problems, I mean, a lot of it is about delicate issues that we haven't quite nailed down the proofs. We think we know what's true but we haven't been able to prove it. So to me as an applied mathematician some of these things are less interesting. They're like mopping up operations but, you know, they help us understand from beginning to end why something is true. Some of them are problems with the flavor I talked about, with complex systems, enormous numbers of variables. You know but I think the future may be more in computation, I'm afraid to say. That calculus is and its… its style of continuous mathematics is giving way, it seems, to machine learning and linear algebra and data science and all that. That seems to be the most exciting part of what I can see in applied mathematics today.

BH: Asking you this questions probably a bit unfair because you're like a popularizer of mathematics and you're an applied mathematician but I'll ask anyway. And that is, how do you feel about the fact that when you're trying to inspire interest in calculus or mathematics in general you always have to go to… applications. Like when I first asked you to explain calculus t me you didn't talk about area under curves or anything you started talking about, you know, mobile phones and satellites and things like that.

SS: Yes.

BH: Do you feel like this is just like a necessary evil…

SS: [laughs]

BH: …or do you feel that it is the best thing about it? 'Cause I sometimes feel like no, I wanna hear about the beauty of the mathematics. That's what gives me goosebumps sometimes when I see something mathematical that I understand. I think, wow this is amazing. And yet, I see so many other people and you all have to do it so often saying, let me talk to you about satellites and going to the moon and like… I know [stutters] I don't wanna hear about that I want to hear about the mathematics.

SS: [laughs]

BH: But you think you can't go down that route?

SS: Well, okay, I'm sure there are some listeners who are cheering right now for you because I hear this from many people that mathematics is magnificent on its own and why cop out with these applications. We don't need that. I agree with you. You can make that case and I… half of me agrees with you. [laughs]

BH: You're a mathematician after all.

SS: I am a mathematician and I love those parts of what we do and these ingenuity of the arguments, the elegance of them, the human creativity, all of that is there in pure mathematics. Forget about the applications. So in each chapter I try to show examples of that kind of ingenuity and as you say it stands on its own and in the same way that fine art… you know I mean you learn to read not just so you can read street signs but so you can read Shakespeare or the best, you know, poetry. These things have worth in themselves. Certainly mathematics does. And I get that. I think for my reader that is not gonna be as easy a case to make. And as you say I am an applied mathematician and in my heart I don't only want to make the pure mathematical argument. But I also don't wanna neglect it. I try to give fair… you know, give it a fair shake. But I think the pure mathematicians are sometimes a little narrow, that they don't… I mean… if it were just the beauty of calculus, that's misleading. It has changed the world. Why are we afraid to talk about that? We're not selling out to talk about saving lives of HIV patients or people who need facial surgery so that now they can, you know, get rid of congenital malformation in the way that they look, that's very important to many people. Getting to the moon was important. Nuclear weapons and what they, you know, what we have done to the world. Calculus has not always been a force for good. But it has had world changing impact and we might as well tell the truth about that. You know, it's like one thing to talk about something I'll just throw out some jargon, the p-adic numbers. The p-adic numbers are pure mathematics that the great mathematician Paul Erdös said once to some other mathematician, you know they're not really real. Which is kind of funny for a pure mathematician to say about a mathematical concept like the p-adic numbers, they're not really real. Well, really none of mathematics is real. It's all made up. I mean it might have some mirror to reality but it's all… conceptual. It's all in our minds, and yet for someone like Erdös the p-adic numbers seemed a little too contrived. They seemed too artificial for him. Alright, so I mean I think if we only make the case for calculus that it's a marvelous way to find areas of curved shapes. First all that's a lie. That's not all it is and that's not really why it matters. In my heart I think I do believe the applied reason [chuckles] is the reason that this matters most but I certainly love the beauty of being able to find the area of a circle. That is not about finding how to tax the poor farmers whose field happens to be circular, it's a case of… the human quest for understanding like a lot of pure mathematics is. So that should be celebrated too. I agree with you.

BH: I always wonder though when I hear people talking about, you know, like when you're being the cheerleader for calculus which you are at the moment which is fair enough.

SS: [laughs]

BH: How much credit do you get? Like, oh we wouldn't have got to the moon without calculus. We also wouldn't have got to the moon without rocket fuel or we…

SS: Correct!

BH: We wouldn't have got to the moon without the bread that the people were eating who were building the rockets and I'm wondering who gets how much credit and…?

SS: Right it's a very fair point. Because you could say we wouldn't have gotten there without the addition symbol.

BH: Yeah. [laughs]

SS: Right? I mean we need to know how to do everything in mathematics, why are you singling out calculus? And I…

BH: Yeah.

SS: Okay, rather than concede that point though, although I accept that there's a lot of merit to the point, there is something uncanny about calculus as calculus which is if we look at the way that the laws of nature are expressed at the fundamental level in every field where we know those laws whether it's the motion of objects, whether it's even quantum mechanics where we can't speak of classical trajectories but only probability waves, well Schrodinger's equation for the evolution of probability waves is a differential equation. It is phrased in the language of calculus. So even though Newtonian mechanics doesn't work in quantum theory, Newtonian calculus is still the language of choice. That's a pretty spooky and interesting thing. And so this is an big claim for me, you know, in the subtitle of the American version of the book, How Calculus Reveals the Secrets of the Universe, that's what I'm talking about. The language of the universe meaning these fundamental laws, whether it's motion of water or air or heat or objects like I say flying around or quantum waves… they're all… governed by differential equations. And they're not governed by linear algebra, they're not governed by group theory, I mean I know some people disagree and say yes group theory is fundamental to particle physics but I still think… if you had to pick one branch of mathematics that is the language of the universe it would be differential equations, and that happens to be a part of calculus. So I… I really do think… you know, maybe it's dumb to single it out. People have asked why not just say mathematics is the language of the universe like everyone else would say? Yes, okay, we can do that but I think it's a sub-dialect. That is the very specific language, it's differential equations. So I'm… [chuckles] it happens to be where my research is, you know, I guess I'm very parochial about that but I think a case can be made. But I also don't wanna oversell it, having just oversold it, [chuckles]…

BH: [laughs]

SS: Yes! We need, you know, we wouldn't have… as you say we wouldn't have put astronauts on the moon without many other things. And so calculus has always been a supporting player. It's never been the star of the show. But I think too long people didn't even recognize that it's been in every movie. You know in medicine, when you go for a CT scan, there's calculus under the hood. So I just want that to be recognized. It's a supporting actor that doesn't get enough credit.

That’s really the honest truth.

BH: Alright, well I feel like… I’ve kept you away from looking at these papers for too long now. [gentle music fades in]

SS: [laughs]

[music continues]

BH: So we should probably stop recording at this point and you can actually get back to… weeping over the… [chuckles] [music continues]

SS: [laughs]

BH: Weeping over these documents. [laughs]

[music continues]

SS: Thank you, Brady. I’ll begin blubbering now.

[music continues]

BH: Now that’s it for today but after the audio recording I did break out the camera and I filmed a bit of Steve checking out these documents and talking to Keith Moore from the Royal Society. So see the show notes for a link to that. And thanks to Keith and the Royal Society for hosting us. [music continues] All I’ll also link to an Objectivity video we made a little while back with Hannah Fry where she’s also looking at some of these documents and having bit of a chat about them. Steve’s book, it’s called Infinite Powers, I’ll include links so you can buy a copy and in case you’re wondering Steve also did donate one to the Royal Society collection, so it’s there for posterity. [music continues] Thank you to the Mathematical Sciences Research Institute, which supports the Numberphile podcast, and also the audio engineering company in Berkeley, California called Meyer Sound which sponsored this episode. We really appreciate them. [music continues] I'm Brady Haran and I'll be back again in your podcast feeds pretty soon but until then, do feel free to rate the show or leave a review. If you say nice things about us it helps the show grow and ensures we can make more of them. If you wanna say bad things about us, well, well maybe don't leave a review in that case. I'm also surprised you've been listening for this long. [music continues] To check out more Numberphile videos, find out more information what we're up to, maybe support us on Patreon, go to Numberphile.com, we've got everything there you wanna look at. And… well… bye for now!

[music gets louder and then fades out]