# Numberphile Podcast Transcript <br> Episode: The Mathematical Showman - Ron Graham (1935-2020) <br> Episode Released July 142020 

Tributes to mathematician Ronald Graham - a man of many talents.
Our guests include Steve Butler, Tom Leighton, and Joe Buhler
$\underline{\text { Ron Graham 'fan' webpage }}$
Ron Graham papers
Ron Graham on Numberphile
A collection of Numberphile videos about Graham's Number
Tom Leighton written tribute to Ron
Steve Butler tribute to Ron

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[Gentle music box plays]

Brady Haran [BH]: Hello, I'm Brady Haran, this is the Numberphile podcast and today's episode is a tribute to the mathematician Ron Graham who recently died at the age of eighty-four. Ron was a giant in the world of mathematics and we'll be talking to various people who knew him well, but I wanted to start with my own story about Ron Graham. That's because without realizing it, Ron was a real inspiration behind Numberphile itself and I'd like to explain why. See as a boy growing up in Australia I was really obsessed with books of facts, these were things I'd get at a local library, and page after page of trivia was listed and one thing that was common in many books at the time was this number called Graham's Number. It was the biggest number ever. Well, the biggest number used in a proof. But that was a minor detail to me at the time. I was obsessed
with the enormity of this number. I remember reading the fact that if all the atoms in the universe were turned into ink, you still couldn't write down Graham's Number. I recycled this fact all the time. I would tell my friends, I would tell my family. I don't know why it was called Graham's Number, I think I assumed the person who had discovered it was called Graham. I probably thought Graham was the first name like Mike or Jane or Bill. I think it was much later in life I realized Graham was a surname. The chap was called Ron Graham. I didn't know who he was, where he was from, anything like that. I just knew he'd come up with a really big number. A bit later in life when I was old enough to be recounting stories to children I would tell them about Graham's Number. I'd use that ink story time and time again. The kids would be wide eyed, that's such a big number! I would use it to inspire an interest in mathematics and numbers just as it had inspired my interest in mathematics and numbers. Role the clock forward a bit more and a chance came to start a Youtube channel about mathematics. And I decided to call it Numberphile, and focus it on numbers. And it really was because of Graham's Number. That one number that had inspired me for all these years, I thought wouldn't a Youtube channel all about different numbers be fantastic? Of course the channel has evolved over time, but that's where it started. One of the early videos we made on Numberphile was about Graham's Number. It featured Matt Parker, Tony Padilla... Tony came up with a new fact about Graham's Number. That if you encoded all the digits into your brain, your head would collapse into a Black Hole. It was a new Graham's Number fact to replace the ink one, I guess. Anyway after that, a collaboration started between Numberphile and the Mathematical Sciences Research Institute, MSRI, it's a prestigious place in Berkeley, California and I was talking one day to the director, David Eisenbud, who knows everyone in mathematics, I told him about my obsession with Graham's Number and he says to me, oh I know Ron Graham, I've got his phone number here, why don't I call him? Perhaps you could go and meet him and interview him? And I have to say the idea of meeting Ron Graham had never occurred to me. I have to say I was pretty excited and not long after that I was on a plane to San Diego and knocking on Ron Graham's front door. I couldn't believe it. I don't often get nervous or starstruck but this
was one of the few times I was a little bit nervous. Ron opened the door and it was amazing. He was an amazing man. Totally not what I expected. Sure, you know, he was a bit older, but he still had kind of long blonde hair, and he was quite trim and athletic. He was great company, showed me around his house told me stories, loved talking about juggling, loved talking about Starbucks, loved talking about his wife who he clearly adored. We did then some filming, I'm afraid to say I made him talk about Graham's Number probably way more than he liked to but he was completely accommodating. We did a bunch of other videos, he took me out to the garage, he showed me a bunch of Paul Erdös files that he had stored there. He was so generous with his time, when I thought my time was finally up he said, do you wanna go for some food? Can I take you for a meal? I couldn't believe. Ron Graham was gonna take me out for a meal. We went to a sushi restaurant, Ron drove in his car which has the license plate NUMBER, which I loved. We sat there and talked about mathematics, now I'm no mathematician but Ron pitched it right for me. He pulled out some napkins and scribbled down numbers as mathematicians so often do. I'm not ashamed to say I souvenired those napkins and still have them to this day. Then kindly Ron drove me back to my hotel, we parked out the front, I said my goodbyes and got out and then as Ron drove away the doorman at the hotel said to me, how was your day? And I said, see that car there, the one with NUMBER written on it, pointing that license plate, and the doorman said, yep. I said the guy in that is a famous mathematician. He came up with the biggest number ever and if the whole universe was turned into ink, you still couldn't write down the number that's how big it is. The doorman looked at this Lexus with NUMBER written across the back, driving off into the sunset and just said, that's pretty cool.
[Gentle violin music]

BH: And before we continue let me just point out a few background points, you might hear people speaking today of Fan or Fan Chung. She's a very accomplished mathematician in her own right but also happens to be Ron's wife. You'll also hear people speaking of the legendary Paul Erdös, this larger than life
vagabond mathematician who collaborated widely and lived a strangely nomadic lifestyle. Ron was a really important figure in Erdös' life and kind of acted as a business manager for him, handling all things like money and other worldly issues which were of little concern to Erdös. Finally no discussion of Ron Graham will be complete without mentioning juggling. Ron was an accomplished juggler and a big part of the world juggling scene. You'll hear more about that. Our discussion starts with a close friend and collaborator of Ron, Steve Butler.

Steve Butler [SB]: So I got to know Ron... well I mean knew about Ron before I really knew Ron, 'cause I read about him in the books about Erdös and I went to graduate school at UC San Diego, I was interested in combinatorics and Fan Chung, Ron's once wife, she was sort of the best and my field of combinatorics and so I started working with her but of course I wanted to work with Ron. And at the least at the very least I wanted to meet Ron, 'cause you know, he's sort of this legendary figure. So, I ran past him in the stairwell once and there was another time I helped Fan and Ron move two houses. They were about two hundred feet apart from each other so it was a really easy... you just carried things out one door into the next. I said, okay, what can I do? And what I ended up doing was of course the classic move, I started erasing the chalkboards. So, Ron taught this Intro to Discrete Mathematics course for about maybe fifteen years and so I started sitting in on the class and then I noticed that before the class started that the boards were messy, people didn't clean up the board so I went and erased the boards and this gave me a great excuse to sort of mild chitchat. And then eventually he mentioned that well he hadn't had his papers online yet. And I said okay, good, I can do that. I can do that. And so I spent... it took me probably about six months of tracking papers down, scanning them in, putting it all together. I even found a few papers that Ron had forgotten about, 'cause you know he had so many he'd lost track of a few of them. So over the course of that six months we got closer and closer, he started feeding problems, and fortunately I was able to make progress and of course Ron, once he knows your strengths, he can start feeding you better problems and really fine-tune.

Like okay this was the good stuff. And so, slowly over time I got to do more and more and I mean... in some sense I was kind of an assistant, a friend, a collaborator and in the end I... I almost became like family.

BH: So it sounds a bit like you sort of at the start you were kind of ingratiating yourself a bit 'cause you wanted to meet the legend and then when he obviously came to know you and developed your strengths you became more of a peer over time, and a family member?

SB: The thing is I figured out later... I went about it the hard way. You see, Ron... he's very generous and if you were just go knock on his door, he would talk to you. You know... he was...

## BH: Yeah [chuckles]

SB: Yes, he was this legend but it doesn't mean he was an off-putting legend. He really was very welcoming and I think part of it was he was sort of too legendary... for his own good. In the following sense, there were a lot of math graduate students who would probably wanna work with him but they were too intimidated to go and talk to him and because he was in a different department they didn't have a natural way... like oh lemme take his class, well he didn't teach a graduate class, and so a lot of students really never tried to approach him. It's one of the reasons he doesn't have that many graduate students when you look at it. You would think he'd have many many graduate students but no no, very few.
[gentle piano music]

BH: Tom Leighton's the CEO of technology company Akamai and knew Ron for over forty years.

Tom Leighton [TL]: I think when I was a graduate student at MIT, met him
probably... maybe when he was hosting a Erdös lecture, he'd bring you know Paul around to MIT, I was fascinated by, you know, the problems they were working on. And then went to Bell Labs for the summer, you know, and this would have been in the late seventies, early eighties and... got a chance to work with Ron more there. And, you know, obviously a very talented mathematician, and he spent a lot of time mentoring young people in the field. And I was one of the people that really benefited, you know, from Ron's mentorship.

BH: Mathematician Joe Buhler was another of Ron's good friends.

Joe Buhler [JB]: We were both into the quite active scene of mathematicians, computer scientists, who juggled.

BH: Right.

JB: And so we both knew of each other.

BH: Was he a really good juggler? [laughs] How good a juggler was he?

JB: Well he could do five balls, he was working on seven, so at that point for an amateur juggler to be at that level was fairly striking. He was never quite at the level of a professional juggler who would put on a particularly slick show but he was very good. He had a net set up in his offices, you probably know, where he could practice juggling five and seven balls in his office.

BH: Why do you think juggling appealed to him so much?

JB: Well, juggling is a cross between pattern, number, physical activity, all these things that Ron was good at and which I liked as well. We both liked music, there's a lot of rhythm to juggling, it just seems to be a... something that a lot of mathematicians, especially at that time, got into. As well as computer scientists, although several people have said that it's sort of follows the
differential equations of an epidemic and Ron was one of the early infected people.

BH: So Steve, after all these years of knowing the name and thinking of him as a kind of mythical figure in your field, what was the reality like? What was he really like as a person, compared to what you thought it may have been in your head?

SB: It's sort of... you think of the mathematical genius. So somebody who spends their days working on problems and they only do math and they're sort of on a different level than us and not really relatable. And Ron is kind of the opposite of that. He shows look you can be a nice person and also a great scientist because he was so generous and warm and kind. And you didn't have to be a mathematician or even you could be a young high school person. In Ron's office, I actually cleaned it out for them, about a week before he passed away. There were two things sort of that stood out and Ron had all these awards over his whole career. Honorary doctorates, lifetime achievement awards from prestigious societies, but he only had one award that he hung up in his office and that was a teaching award. I think he was really proud of his teaching and especially because the teaching award was one that was sort of selected by the students and so I think that he really loved that. The other thing that he had was a sort of a kind of a picture and it said, Another Roof, Another Proof. It was a saying about Paul Erdös, and what's special about that? Well, it was from a high school student. A high school student had written Ron and said you know, I'm very interested and I know you're probably aren't going to respond back to me but, you know, you're very inspirational and then Ron wrote back and actually sent them a book and they corresponded for a while and so Ron got this picture and he put it up in his office. He had this way of connecting with people. I think at heart Ron was an incredible showman and he loved performing. So Ron loved seeing people be amazed and one day I went out walking with him, we were walking across campus and he saw a group of jugglers. And Ron is, of course, former president of the International Jugglers Association and in fact there's a
very famous trick called Mills' Mess, it's a three ball trick, and it's named after Steve Mills. Okay who is Steve Mills? Well, Steve Mills when he was a teenager was growing up in New Jersey and he learned how to juggle from Ron Graham, and in fact Mills' Mess was basically developed by Ron. So Ron had developed all these amazing things. So anyways we were walking across campus, he went up to the students and I was just gonna say this is Ron Graham, he's this famous juggler and Ron was like no, no, no, don't tell them, don't tell 'em, 'cause he wanted to sort of like say well, huh, maybe I can try let's see and then all of sudden start doing this amazing tricks so that they're like, woah! Wow! How did you do that? That's Amazing! That's the performer in Ron. He really liked to perform. And he had a bag, and almost always he always had a deck of cards or maybe some coins or a trick, something he can pull out and show people, or he could pull out his wallet and one of the things he did sort of towards the end of his life, he got kind of interested in... you know the digits on the serial numbers on the bank notes. So he said well look, here's this interesting one, you know, four fives in a row. What are the odds? And you know, all these sort of interesting things he collected, just ready to pull out whenever he wanted and he loved talking to everyone. And he had this amazing ability to remember people, remember details, especially of course all the Starbucks employees, because he spent so much time there. Almost everyday, several times a day. He had quite the coffee habit.

JB: Well, Ron was very socially adept, very friendly, he was very nurturing to young mathematicians, to colleagues, he was just an amazing sort of graceful gentlemanly person to talk to and if you shared anything, you would find yourself having a fairly interesting and sometimes intense conversation bout it.

BH: This flare, this showmanship that Ron have, did that also translate into his proper mathematics?

SB: Well I mean certainly if you look at some of the math that he did, he did... math of magic, math of juggling, I think that showmanship that's certainly an
element. Ron had a way of giving talks that made them memorable. I've had people come up to me and say I remember a talk that Ron gave fifty years ago, and I'm just like, wow! I don't remember half the talks that I saw in the last year, because they weren't memorable. So what would Ron do? Well, he would always think about, okay, it's not just enough to talk about the information, I have to do something to really connect with the audience. So he had all sorts of things. So there was one time when he was using an overhead projector and this was back in the day, they're were two overhead projectors and you'd walk back and forth between them and you'd put up various diagrams. He was giving his talk and then in the middle of the talk he was at one overhead projector and then he said just like what's over on the other side and then he, you'd look at the other one, and there was a hand there. And it's like what, wait, what? Where's the hand coming from? He had a fake hand.

BH: [laughs]

SB: He brought a fake hand and was pointing using the fake hand to this slide and then he moved across the room and of course he didn't draw attention to it right away. So... things like that really set him apart.

TL: You know there's a natural synergy there with you know inventing the mathematics to capture the, you know, the mind to say wow this is really cool and that does carry over into showmanship and you know 'cause that's what gets your attention and interest and you can appreciate the beauty of something that way.

BH: He was a proper hardcore mathematician too, though? He like he was the real deal, you know, with papers?

TL: Yeah Ron was the real deal and yes he... in mathematics it's not always common that the real deal and brilliance in mathematics is in the same person as somebody who can really communicate well and be a showman and really
attract interest, you know, crystalize a beautiful idea in a way that a large number of people like can understand it.

SB: I think what made Ron such a great mathematician is he kind of had a way of just working the problem. Breaking things down... he did this with everything in life, 'cause he said, lemme break things down, and let me try to figure out what's the hard part. Where are we getting stuck? And he would just work on it, examples after examples, do computations, look at data. He was really good at looking at data, seeing patterns, and on his table in his house was a little tiny sign that said, Never, Never, Never Give Up! [pause] And I think he was persistent. He was good at thinking about problems and just working it. Of course, often times at the end, it's like, okay now we have a solution we' ve gotta go back and clean up the proofs that got us from point $A$ to point $B$ but he was just had a kind of this persistence about him and ways of thinking about things in different ways.

JB: Well he's a really distinctive figure in mathematics, partially because of what things he's done in mathematics. What his number of papers is in the at least the hundreds, probably more than that actually. And, if you had to name important people in combinatorics he would be one of the very few names who would approach being at the top of the list. So that legacy alone would leave him a place in history books, but more broadly he played a lot of role in various professional societies, in giving more visibility for mathematics in the broader community, in the large number of mathematicians he has helped and nurtured, in his connection to Paul Erdös, in his connection with the mathematics of juggling, et cetera.

BH: You mentioned that he was the president of a few associations and societies and was involved in those sort of things. Why would that be? Does that... should I take from that that he was like a people person, or that he was a political person, or that he was a popular person? Like, what is it about a mathematician that has them in those sort of roles?

JB: Well he had all three of those attributes. First of all he was very adept at just plain old social skills. Communicating with people, getting them to talk, he was not very sort of... I dunno... self-centered or assuming at all. So he was one of these mathematicians of which there aren't that many... whose social and political skills were at the very high end, but beyond that he was very sort of astute and he... would use that information and connect with people and put people together in ways that are rather unusual. You can find other mathematicians who have similar political skills, some, not so many that have the same social skills, but the combination, the mixture of attributes there is really unique.

SB: One of the things that made Ron so good, and I think this come backs to his personality. I was talking with Fan the other day, that you know, he was very successful in part because he was such a great and friendly person and people like to help him out and share ideas and problems and so Ron had an amazing ability to sort of collect problems, because were willing to share with him, 'cause of how good he was, and how generous he was and then of course he also had the ability to say, alright this would be a good problem for that person to work on. And so he would hand problems to people and so he had this great ability to merge people and problems, the right problem for the right person.

TL: Ron was a... an amazing teacher, you know, and that's, I think really important too. He showed the beauty of discrete mathematics to a lot of people, motivated a lot of people. And it wasn't all just about him proving his theorems, you know? And a lot of times in math the ability to communicate to, mentor other people, teach other people, and, you know, share the magic of it is a rare thing and very important.

JB: There were many things that made Ron unique. One was his focus on combinatorics within mathematics, which was, oh, it was maybe slightly more distinctive at that point and time. He was very friendly, perceptive and famous.

In addition to having a non-academic appointment which gave him a lot of opportunity to nurture and push mathematics as he saw fit.

BH: Would he be like the guy in the room, oh that's Ron Graham? Or was he just one of the crowd?

JB: Well... certainly to anybody who knew he was or was in closely, you know, related areas of mathematics he was a very famous figure, and by of course in the last thirty years Ron Graham is obviously a very well known mathematician. I mean very few people have been president of the American Math Society, the Math Association, involved in the National Academy of Sciences. So he has extraordinary high profile. When I met him... he certainly had a high profile among many mathematicians, but he wasn't an academic and he was working in an area that was a little outside the things that were especially fashionable at that point in time.

BH: What can you tell me about, Ron's attitude to Graham's Number?

SB: I think he liked it. He certainly, you know, Graham's Number is of course an upper bound. And of course everybody just is fascinated by the size of the number, but of course there's a problem in it. In that the problem is alright... you know if you have this certain structure eventually something must be true and Graham's Number is when it was true. Of course when might it be true? Well, maybe as early as 13 . So, there's a bit of a gap. They have brought, of course, the upper bound down quite a bit but still unimaginably large.

JB: Well Ron was adept at knowing what might appeal to what audiences and it certainly a fun question and so yeah I certainly knew about that. Somebody who knew Ron more through mathematics probably would know about that, but not necessarily.

BH: He liked the semi-fame of it though, did he? Or he didn't mind, you
know, being asked about it by people?

SB: Oh no, he loved sharing things. And I think... he had the, of course, a big box he labeled PR. Because he was somebody that people could connect to because, well, you know, maybe you can't connect to high level math but hey here's this person who can jump on a trampoline and do these amazing trampoline tricks who also can talk about magic and who also can somehow connect all that together to mathematics. And so he was wonderful spokesman, in a sense, to say, hey you know, here's this beautiful idea called mathematics and it's not just for some elite people who sit in academic buildings. No, no, math is meant for everybody, and it really is something that has wider appeal.

JB: One of his attributes was his interest in, you know, table tennis. Where his skill was up to low level professional. In trampoline. In juggling. Interested in running, the fact that I like running was something he could immediately connect with and encourage me in doing and so that kind of... devotion to exercise was a distinctive thing and his ability was really pretty high, for many years he could do a one hand hand-stand that requires a great deal of not just balance but strength. And of course juggling was the place where I was most connected to that. So I think his... you know... aptitude for interest in several different kinds of physical activity was one thing, I was going to say. The other is the mathematics of juggling and I wouldn't say Ron and I came late to the party but several people had had a rather curious collection of ideas called side-swaps by some that it took Ron and I, you know, a couple years to really get up on but when we both realized how interesting was not just to mathematicians but to jugglers and the mathematical potential of it, Ron was just very good at nurturing that and encouraging activity in that particular area.

BH: It sounds like he could have been like a professional athlete in another life?

JB: Oh there's no question of that. If he had really decided that table tennis
was where it was at for him and he'd done nothing else, you know, focused on that, I'm sure he would have been quite good.

BH: Steve can you tell me about towards the end? Apparently Ron was still doing mathematics right to the end?

SB: Well, yeah, I mean Ron always did math. There's of course Erdös had this sort of goal, he saying you know what was his ideal way to die? And Erdös said, well I'd be giving a talk and I'd finish the talk and somewhere in the audience he would say well this is really good but what about the general case and Erdös would say I leave that to the next generation [theatrical gasp]. And pass way.

## BH: [chuckles]

SB: That was Erdös ideal death.

## BH: Right.

SB: He came pretty close. Erdös came close. And Ron also came close. So, Percy Diaconis is working on a problem with a student about random walks on the modulo... integers modulo of prime. I can't give all the details 'cause it really is Percy's work but Ron got interested in it.

## BH: Yep

SB: And so he called me up and said, okay Steve what if we changed to prime, what if we did this, okay what happens with these types of primes, what's the difference? You know, how are the quadratic residues distributed? Okay, he'd call me up and ask me a question, I'd run some computations and then he'd call me up and ask me more questions, say okay try this, and then I'd run more computations and that... we did that about four or five times the day before he passed away. And on the day he passed away I came to his house and well he
wasn't up to it... and Fan told me that Ron had something to tell me but wasn't ready yet and unfortunately he never was ready. And so, I'll never know what Ron was going to say, I suspect he probably saw something and said, Ah! I think I understand what's happening, I think I understand why prime's can grow into one, Mod four are different than primes that go into three Mod four, 'cause... well he had a way of seeing things, so, well, now it's up to figure it out.
[pensive piano music]

BH: In 2001 Ron was invited was invited by Tom Leighton to join the board of Akamai, that's a multibillion dollar tech company.

TL: You know Akamai is based on a lot of mathematics and we had our talent pool was highly mathematical, and the problems we were trying to solve were highly mathematical, and so from the discrete side of mathematics and Ron obviously very very smart guy, and that was his area of expertise, so it was a very natural fit for him to be on the board of Akamai. You don't always see mathematicians on corporate boards, I think probably the corporate world would benefit from more but Ron was an obvious fit for us. We're very fortunate he was on the board. Very experienced guy, you know, he had been at, you know, AT\&T, Bell Labs, he understood how the corporate world worked and that's important. You know, he was good at attracting and recognizing strong talent, that's very helpful. He could understand, you know, the problems we were dealing with, the business sense, you know, so all those things make a difference in being on a board.

BH: Did he bring anything to the table different the others?

TL: Well, yeah, Ron [laughs] was different than your typical director. And in a good way and I think a good board is comprised of a lot of diverse talent and perspectives and so you know he did bring something that was different than a lot of traditional directors would bring. And I think that's a positive, a very
positive thing. I think the other directors really appreciated his insights and working with him and found him a very interesting colleague.

BH: He wasn't bring in like the juggling balls or trampolines or anything like that?

TL: Not trampolines! But I think he may have juggled for the board, you know, simple juggling once. I think they were fascinated by... wow there's this brilliant guy here, you know, at the same time he's a world class juggler? How is that possible?

## BH: What will you miss most about Ron?

JB: Boy, Ron is just somebody I could have fantastic conversations with, either in person or on the phone and I'm going to miss his presence. Miss his advice. Miss our mathematical interactions. I can't [chuckles] it's hard to stop [chuckles] with this list. It's just... you know... I knew he was declining but when I got the news it was still... just a real shock. It's a big change in... well not just my life but in the lives of many people.

SB: I think I'm gonna miss a lot of his stories. I really enjoyed doing math with Ron, but I think I enjoyed more being with Ron because... he was so giving and he'd always share these amazing experiences. And I don't think anybody has them all. And I suspect that there's some of them that Ron never shared with anybody and that nobody will ever know, and so I'd get to hear these bits and pieces so... as an example he told me about back in the day he would travel to Hungary and well, okay that was in the Cold War, wasn't very easy for mathematicians to necessarily travel at that time. But he was able to travel, so he became sort of a connection. Well, one time he traveled he brought in five programmable calculators. Well, you weren't supposed to take that, you know, 'cause that was advanced technology, but he took it. And he gave them to the various mathematicians there, one of them went to Laci Lovász who is now the
president of the Hungarian Academy of Sciences and he took it and the next day he had written a little program to factor numbers and, you know, just a very small programable calculator, 100 lines of code, but you know, Laci figured out how to do it. And then he gave another one to Endre Szemerédi, okay very famous mathematician, so Ron once asked Szemerédi, okay so what did you do with you calculator? And Szemerédi said oh, I can't remember, I probably sold it on the black market.

JB: One thing that gives you a taste of Ron as a mathematician and his breadth despite coming from combinatorics was that he was this... is this sort of person... was this sort of person that at the end of a math talk or a dissertation exam, could always come up with a very appropriate insightful question even if he was not very much not maybe an expert in the topic of the talk or the dissertation.

SB: He's not just this mathematical genius, he's also a very kind person and like... one of the things I've learned from knowing Ron, was just how amazing of a husband he was. He was married to Fan Chung in 1983... and I think they were... probably had one of the best marriages I've ever seen. He was so caring right to the very end. Even in his last few days, he kept thinking about Fan. So... [long pause] sorry, that's getting a little bit emotional there.
[gentle piano music]

BH: It seems fitting that last word today will go to Ron Graham himself. I mentioned he appeared on Numberphile and did a handful of great videos on various topics. I'll link to them in the show notes in along with all other sorts of useful resources but now here's a few words from the man himself captured on that day of filming.
[gentle piano music]

Ron Graham: In some sense it... it captures the gap between what seems to be
really true and what you can prove. And often the gaps are tremendous, you can prove something must happen by some time but really it probably happens much sooner it's just that you can't get your hands on that much sooner, so at least you know it happens sometime, that's something and now the idea's to try to... I mean... there's a nice example of that that happened oh within the past year on the Twin Prime Conjecture. This was an old problem that goes back to the time of the Greeks as far as people know. You look at a prime number, that is a number that has no factors except one and itself, such as thirteen or twentynine, and you'll notice that pretty often you'll find pairs of prime numbers that differ by two. Like eleven and thirteen, or hundred and one, hundred and three, for example. And the conjecture was that this happens infinitely often, you'll find pairs of primes that different by two as large as you like and it seemed to be completely unattackable until this recent breakthrough by a Chinese mathematician living in New Hampshire named Jan I Tang who showed that well maybe two... we don't know about two but definitely infinitely often you get two primes that differ by at most seventy million. We say, well wait a minute that's pretty far from two. Well, not really. At least it's a bound. The first actual bound and now people once they understand his ideas, his techniques, have been able to chip that down and now it's down to two hundred and seventy. They think that that's... has about run out of steam, they get any new ideas but it just... kind of reaffirms your belief... not that anybody doubted anyway that these pairs of twin primes do exist forever, so chipping making at least there's some bound, seventy million and once you get that now you take one more step up the mountain and eventually you hope to get to two. You know, a lot of harmless looking questions about the digits of numbers that don't have so much deep number theory information but they're questions people can ask, it's you know for example, nobody knows that if you look at the decimal expansion of pi, that eventually the digit nine doesn't appear anymore. [pause] Right? It could happen that pi goes out and then... $3.1415962 \ldots$ and then at the trillionth digit all of a sudden you don't see nine ever again. Well... c'mon [music cuts in and cuts out] that's not possible, right? But nobody can prove it couldn't happen. Well, it might happen, I dunno [music slowly fades in] well what you think is for pi each
digit occurs about a tenth of the time, ten digits, why should nine be any different from eight, or six? Well, you know, can't prove it.

> [gentle music swells and fades out]

